Institute for Quantum Computing

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On the inefficacy of Gaussian regenerative amplifiers for quantum optical communication

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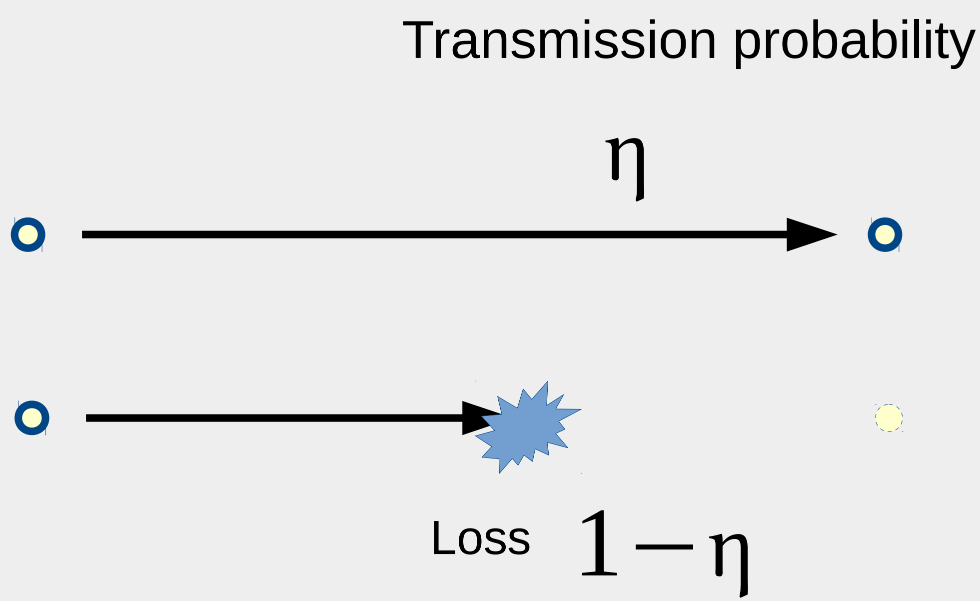
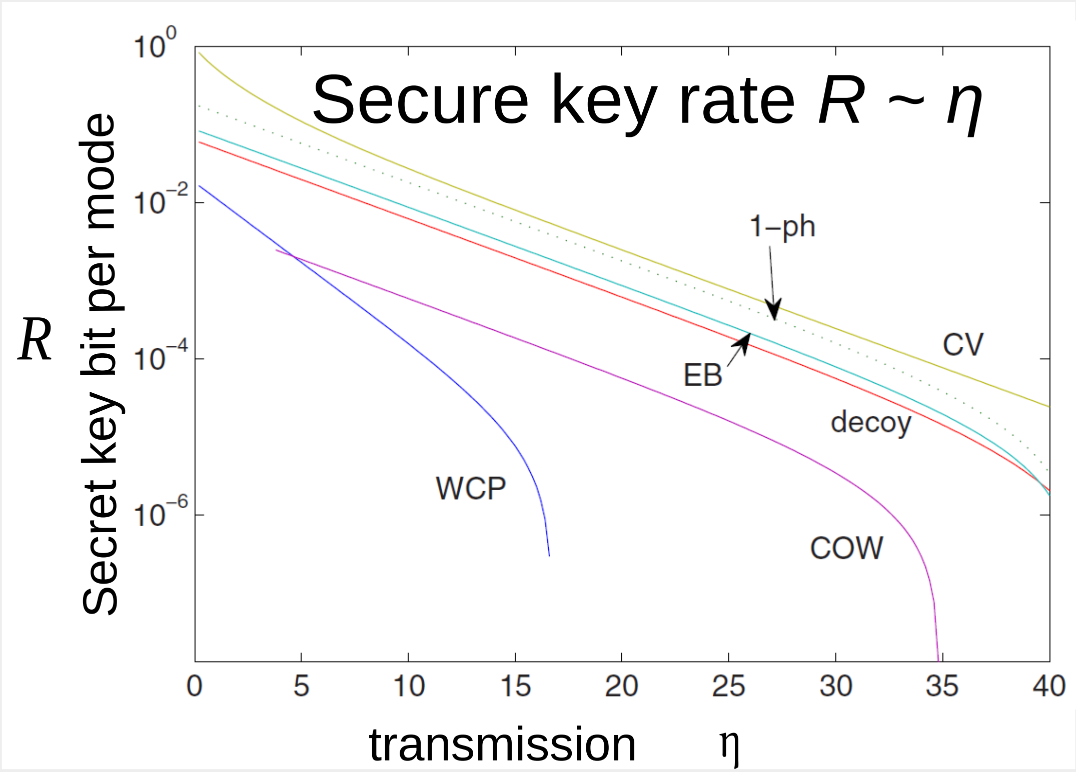
DARPA Quiness Program
Office of Naval Research

Introduction

- The photon loss limits long distance QKD.



Introduction



Scarani et al., Rev. Mod. Phys. 81, 1301 (2009)

- General unpper bound on the secure key rate $$R \leq \log_2\left(\frac{1+\eta}{1-\eta}\right) \sim 2.88\,\eta \quad (\eta \ll 1)$$
- Exponential decay with distance $$\eta = e^{-L_{tot}/L_0}$$

Takeoka, Guha, Wilde, arXiv:1310.0129; IEEE Trans. Inf. Theo. 60 4987 (2014).

Introduction

- Quantum Repeater

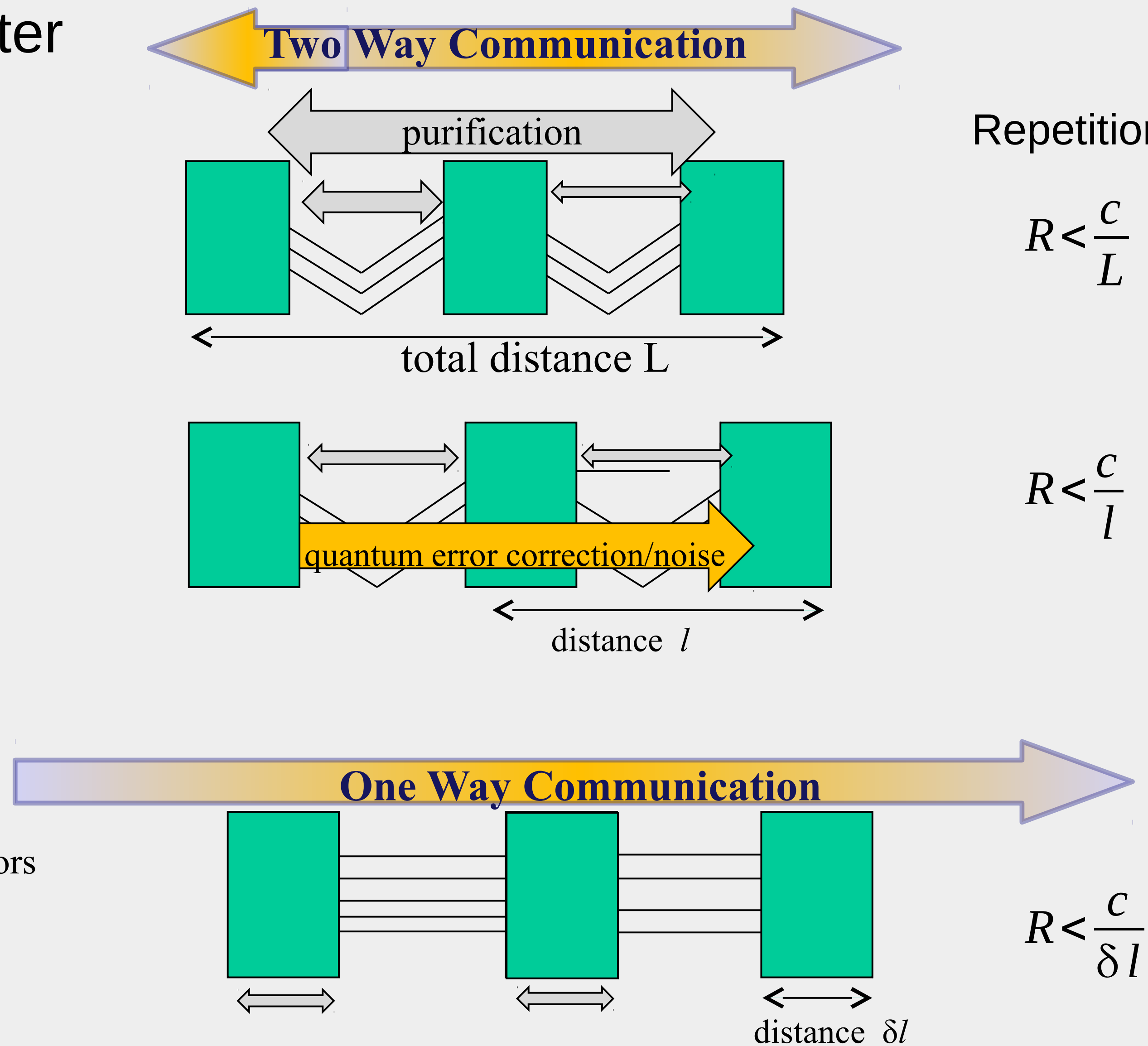
1st generation

2nd generation

3rd generation

QECC for operational errors

QECC for loss



Classical communication

Repetition rate

$$R < \frac{c}{L}$$

$$R < \frac{c}{l}$$

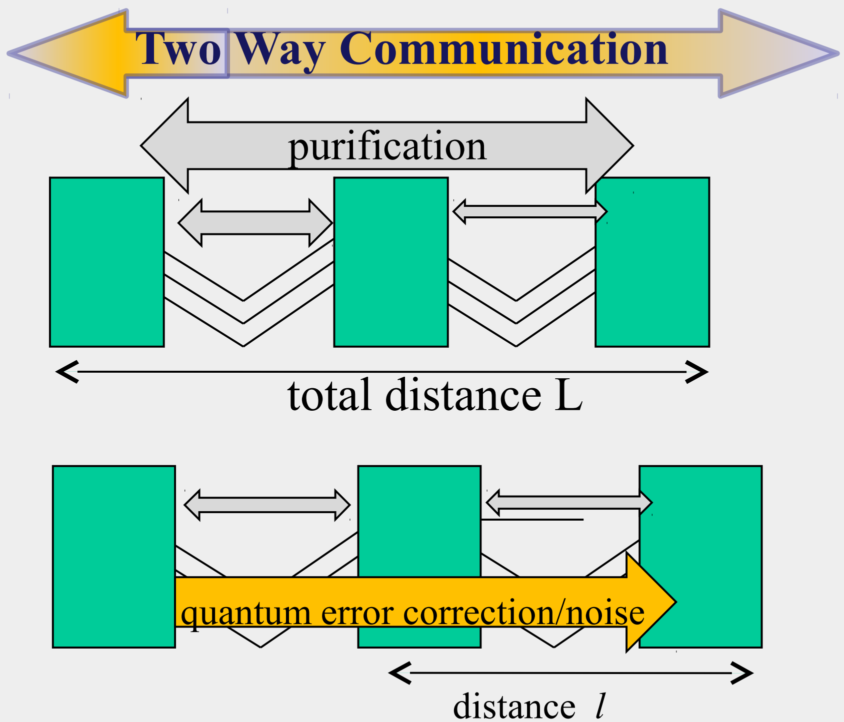
$$R < \frac{c}{\delta l}$$

Muralidharan, Kim, Lütkenhaus, Lukin, Jiang, PRL 112, 250501 (2014)

Introduction

Classical communication

- Quantum Repeater
  1st generation



Repetition rate

$$R < \frac{c}{L}$$

2nd generation

$$R < \frac{c}{l}$$

3rd generation

QECC for operational errors

QECC for loss

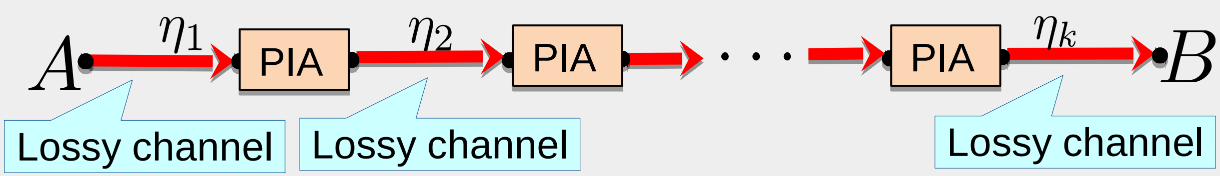
$$R < \frac{c}{\delta l}$$

Chain of lossy channels and black boxes

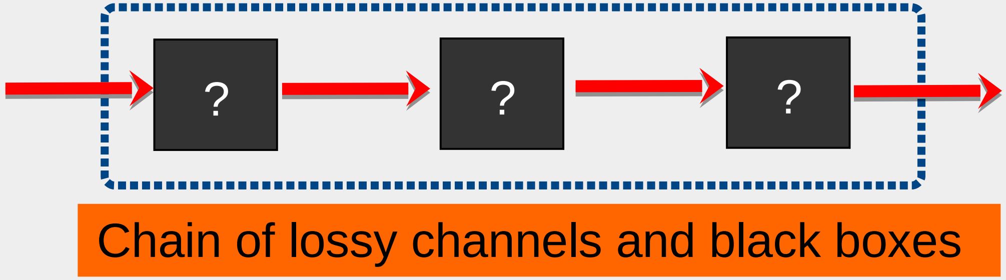
Muralidharan, Kim, Lütkenhaus, Lukin, Jiang, PRL 112, 250501 (2014)

Introduction

- Classical Communication
  Phase insensitive amplifier (PIA) extends transmission distance



- Quantum Communication:
  3rd generation quantum repeater



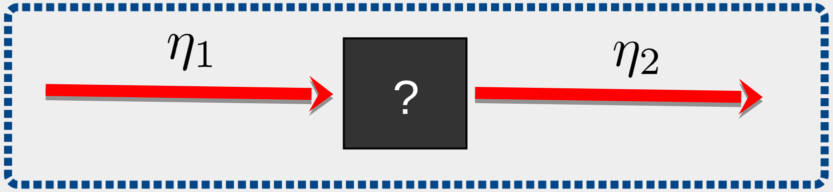
What could those black boxes be?

Single center station between lossy channel

How does a center station modify the total channel?

- Simplest tool box: Gaussian operations

- Could phase insensitive amplifiers (PIA) or generally Gaussian quantum channels work as a quantum repeater?



Chain of lossy channels and black boxes

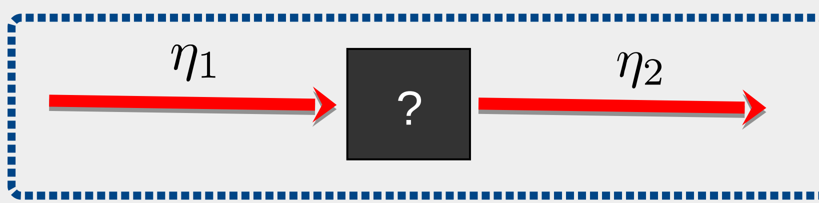
Single center station between lossy channel

How does a center station modify the total channel?

- Simplest tool box: Gaussian operations

- Could phase insensitive amplifiers (PIA) or generally Gaussian quantum channels work as a quantum repeater?

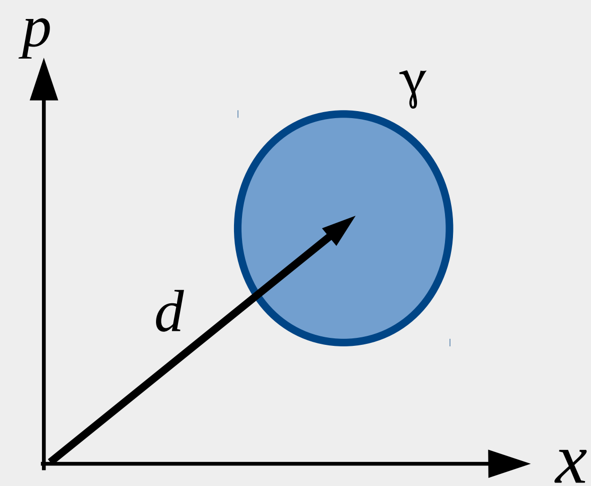
No!



Chain of lossy channels and black boxes

Gaussian states

Quantum states of Light: harmonic oscillators



n-mode bosonic field

- Displacement vector

$$d = \left.\begin{bmatrix} \square \\ \square \\ \square \\ \square \end{bmatrix}\right\} 2n$$

- Covariance matrix

$$\gamma = \begin{bmatrix} \square & \square & \square & \square \\ \square & \square & \square & \square \\ \square & \square & \square & \square \\ \square & \square & \square & \square \end{bmatrix} \quad 2n \times 2n$$

$$\Delta x \Delta p \geq 1/2$$

canonical uncertainty relation

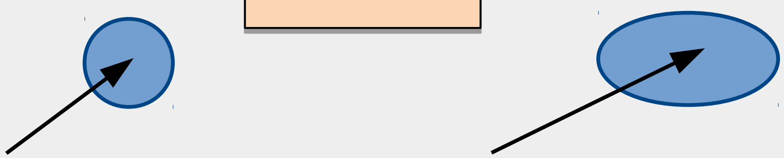
$$\gamma \geq \frac{i}{2}\sigma, \qquad \sigma := \begin{pmatrix} 0 & I_n \\ I_n & 0 \end{pmatrix}$$

Identity matrix: I_n

Gaussian channels

Def. Transform Gaussian states to Gaussian states

$$(\gamma, d) \longrightarrow \boxed{(K, \alpha)} \longrightarrow (\gamma', d')$$



(K, α) = (Gain, Noise)

$$\begin{cases} d' = K^T d, \\ \gamma' = K^T \gamma K + \alpha \end{cases}$$

$$K = \begin{bmatrix} \square & \square & \square & \square \\ \square & \square & \square & \square \\ \square & \square & \square & \square \\ \square & \square & \square & \square \end{bmatrix} \qquad \alpha = \begin{bmatrix} \square & \square & \square & \square \\ \square & \square & \square & \square \\ \square & \square & \square & \square \\ \square & \square & \square & \square \end{bmatrix} \qquad 2n \times 2n$$

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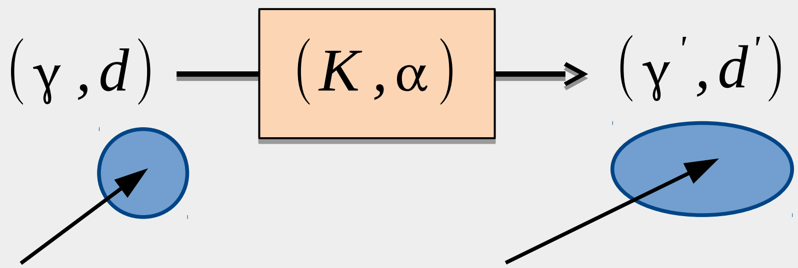

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canonical uncertainty relation

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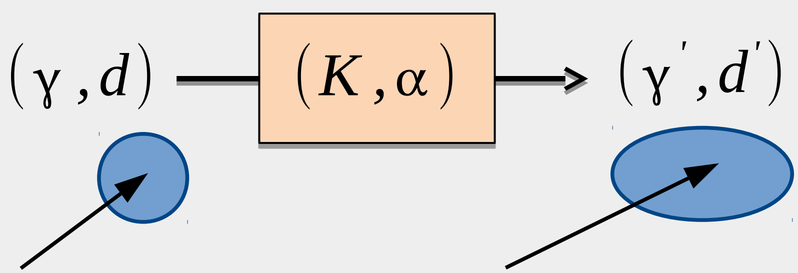
Physical Condition for channels

$$\alpha \geq \frac{i}{2} (\sigma - K^T \sigma K)$$

Gaussian channels

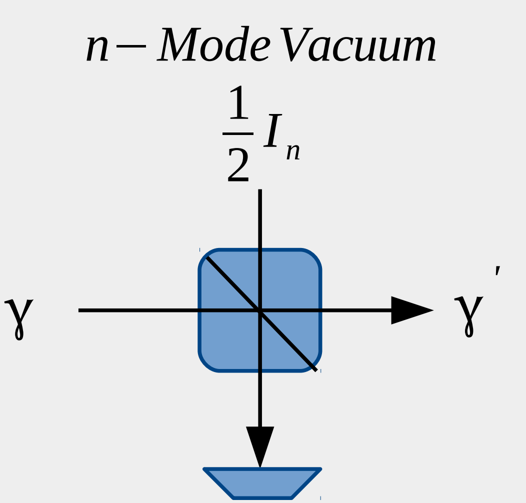
Def. Transform Gaussian states to Gaussian states



(K, α) = (Gain, Noise)

$$\begin{cases} d' = K^T d, \\ \gamma' = K^T \gamma K + \alpha \end{cases}$$

- Multi-mode Pure lossy channels; transmission *η*

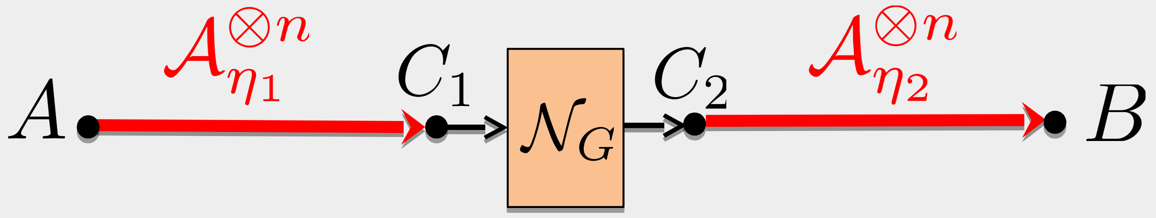


$$K = \sqrt{\eta}\, I_{2n}, \quad \alpha = \frac{1-\eta}{2} I_{2n}.$$

$$\begin{cases} d' = \sqrt{\eta}\, d, \\ \gamma' = \frac{1-\eta}{2} I_{2n} + \eta\, \gamma \end{cases}$$

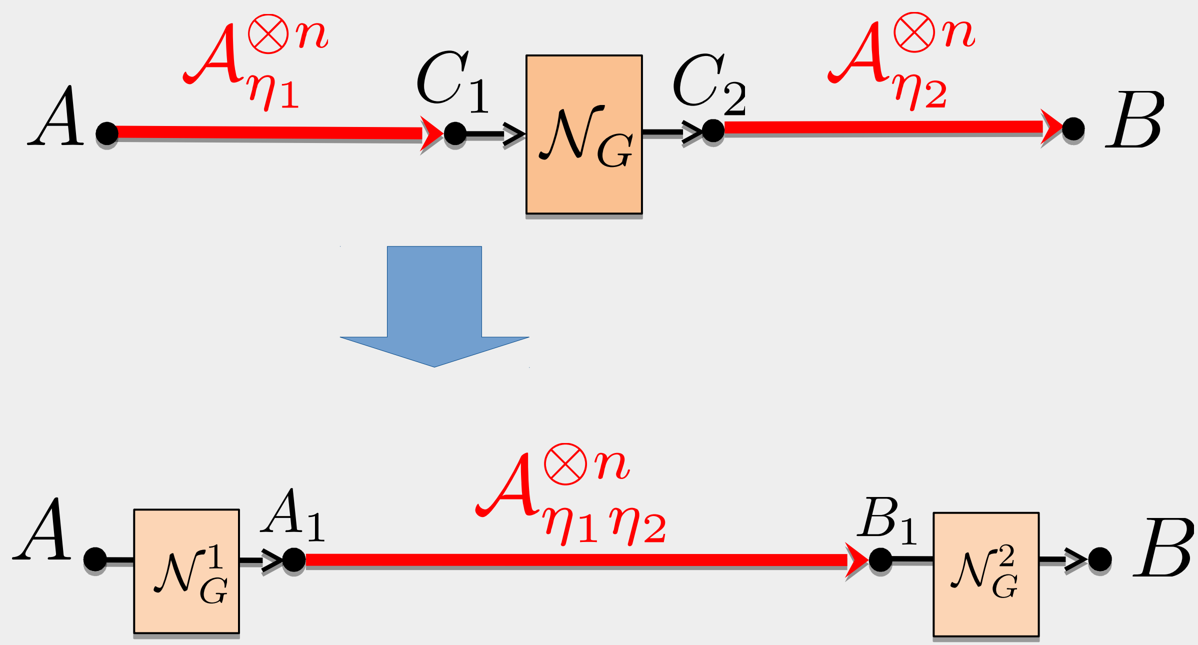
Setting

- Gaussian channel sandwiched between lossy channel segment



Main theorem

- Decomposition of Gaussian center station



Main theorem

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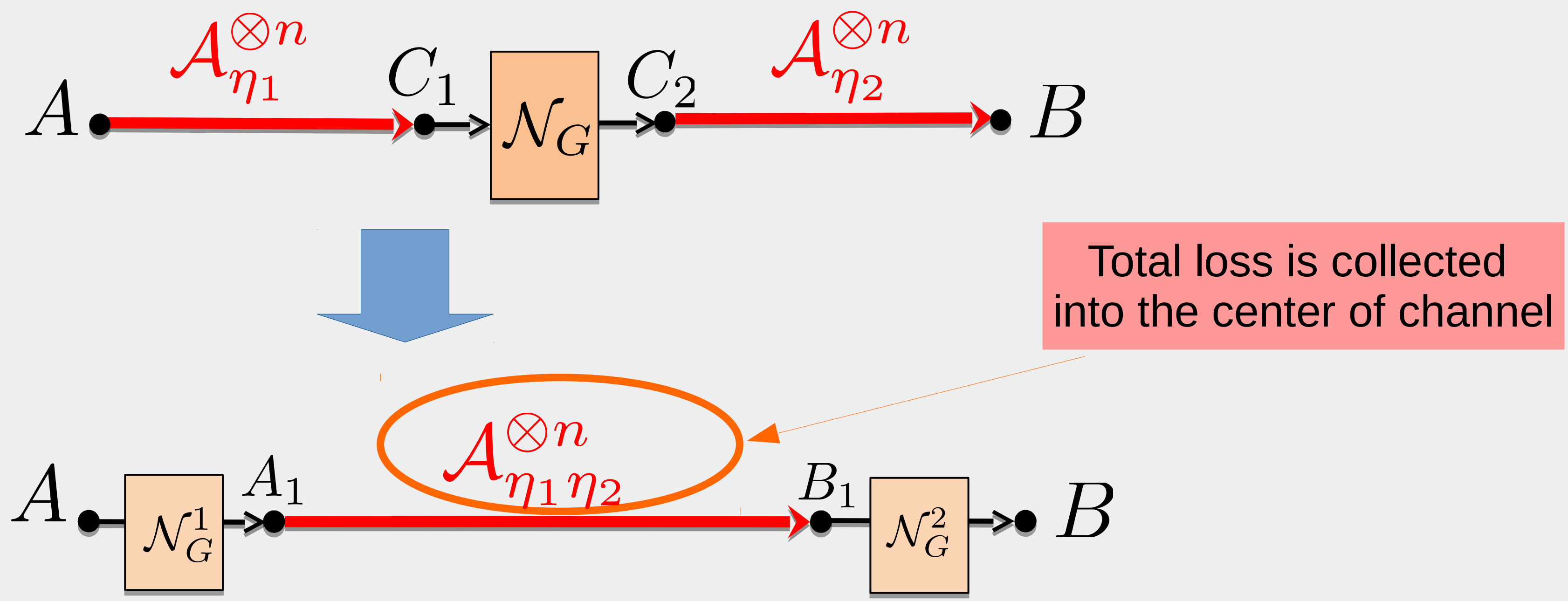

Main theorem

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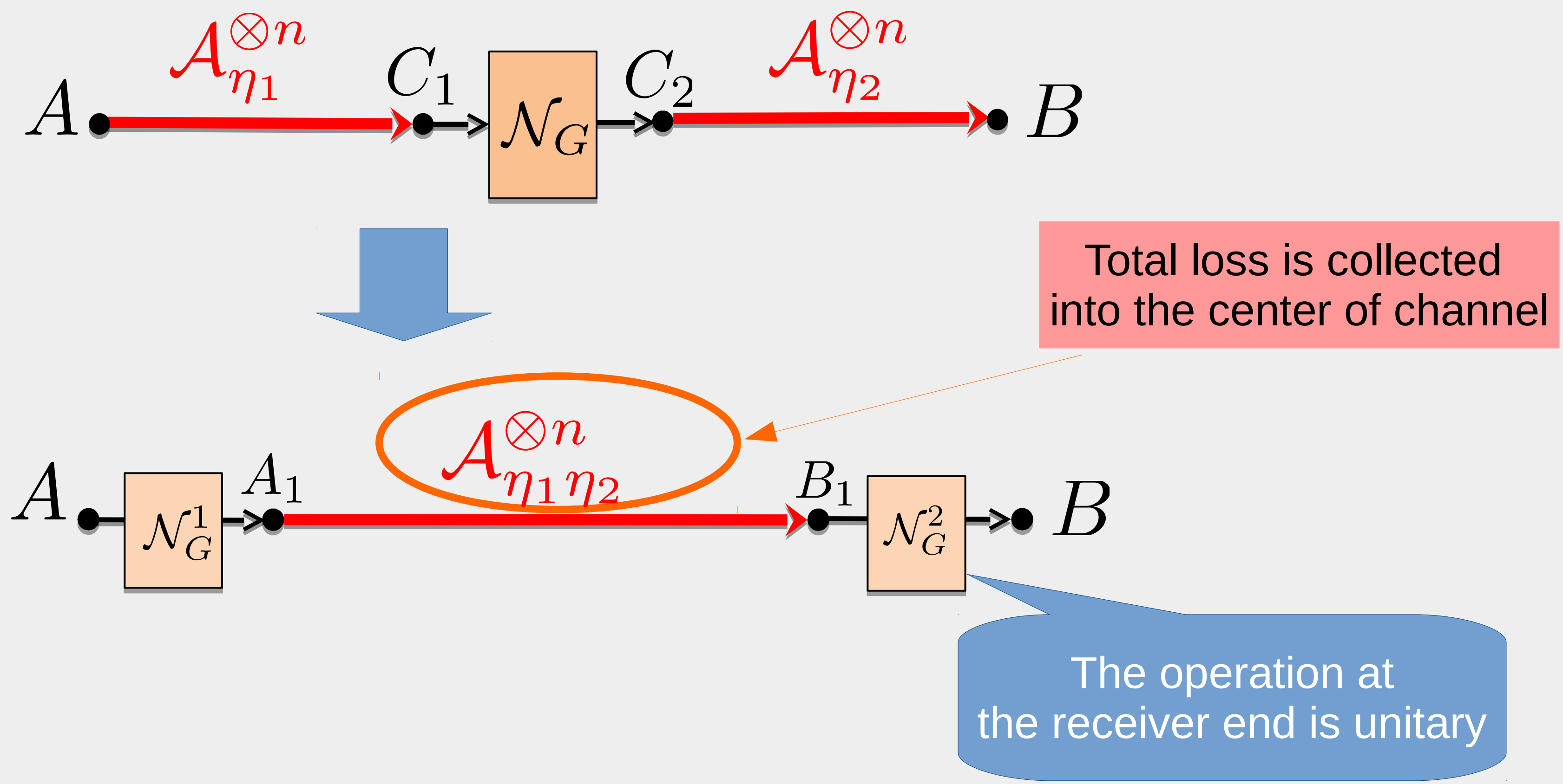
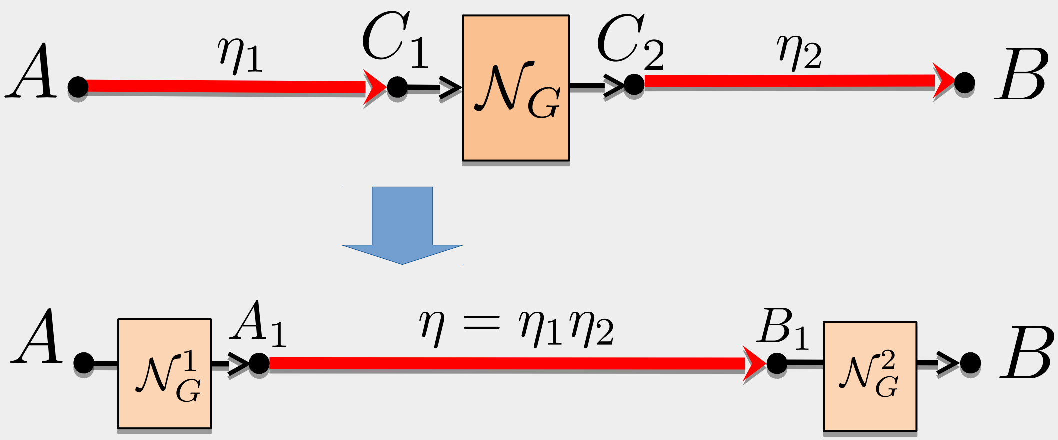
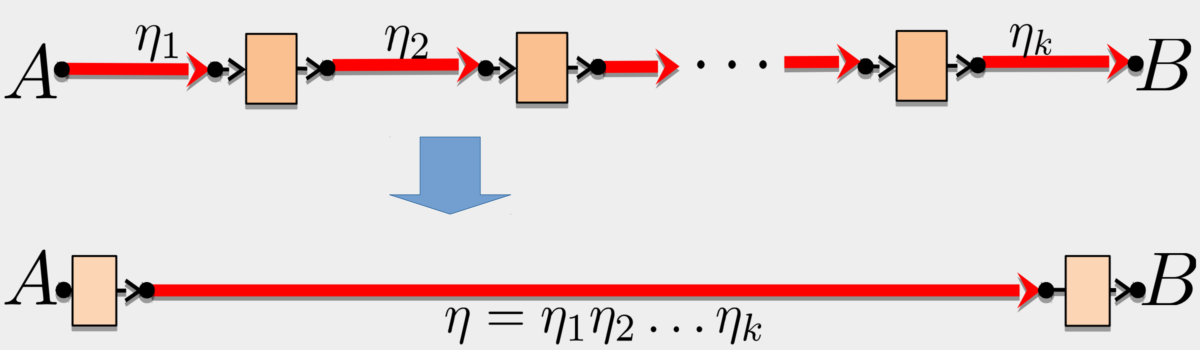


Implication for many stations



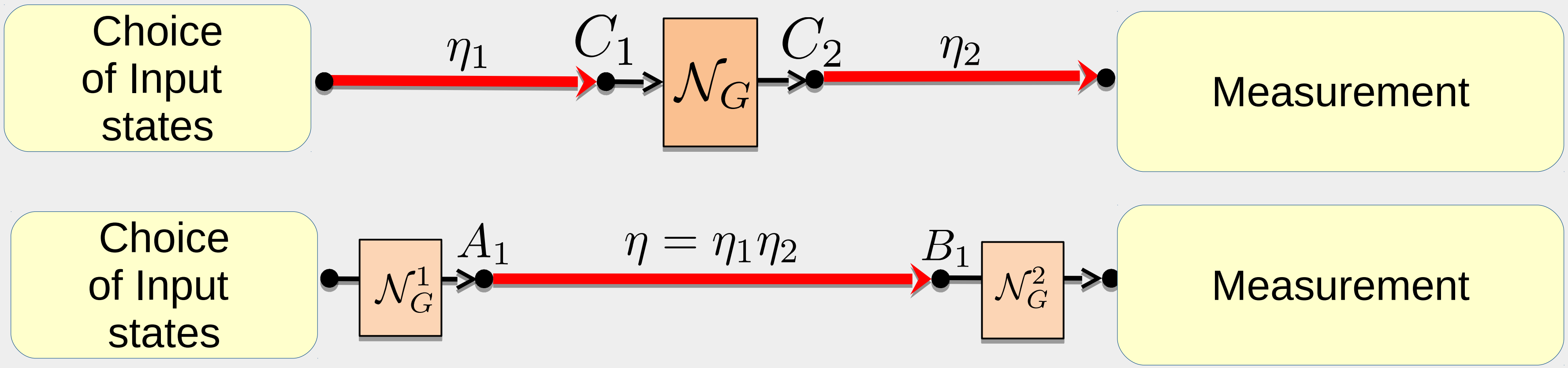
-Effect of loss cannot be reduced!
-Cannot be improved by Interspersing many stations!



★ No difference in Gaussian or Non-Gaussian input states

No-Go result for Gaussian repeater

- Center station → Modification of transmitter and receiver.



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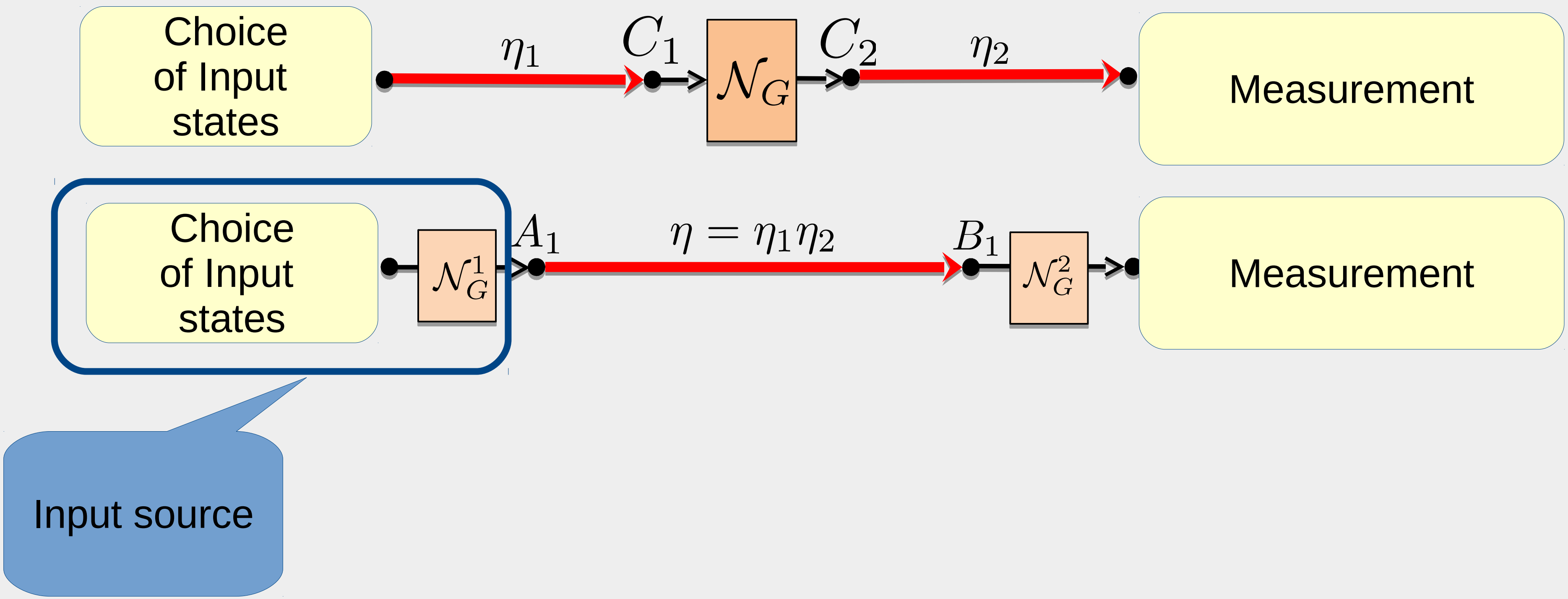

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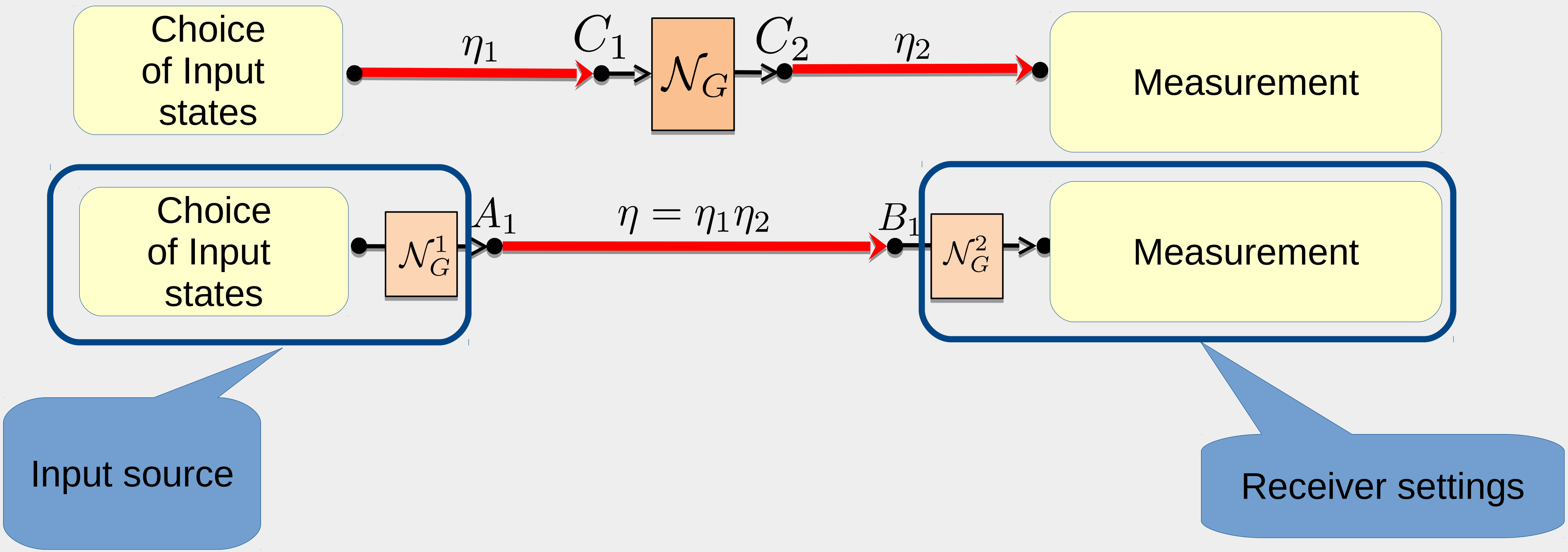

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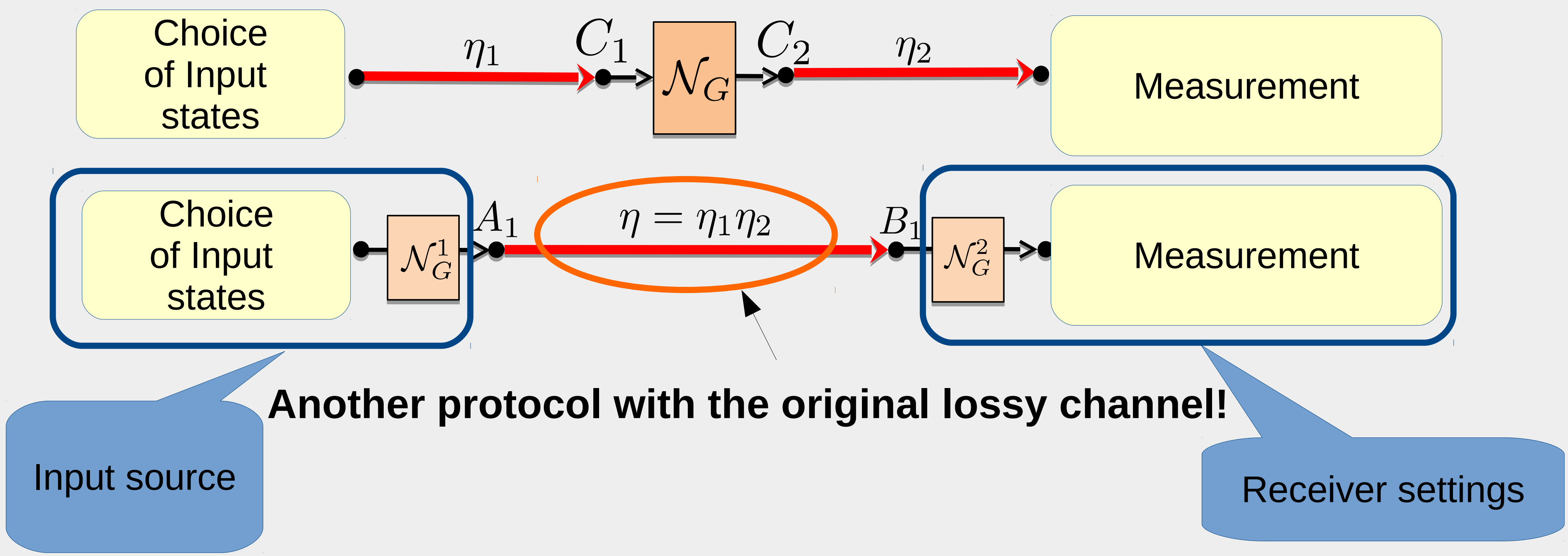

No-Go result for Gaussian repeater

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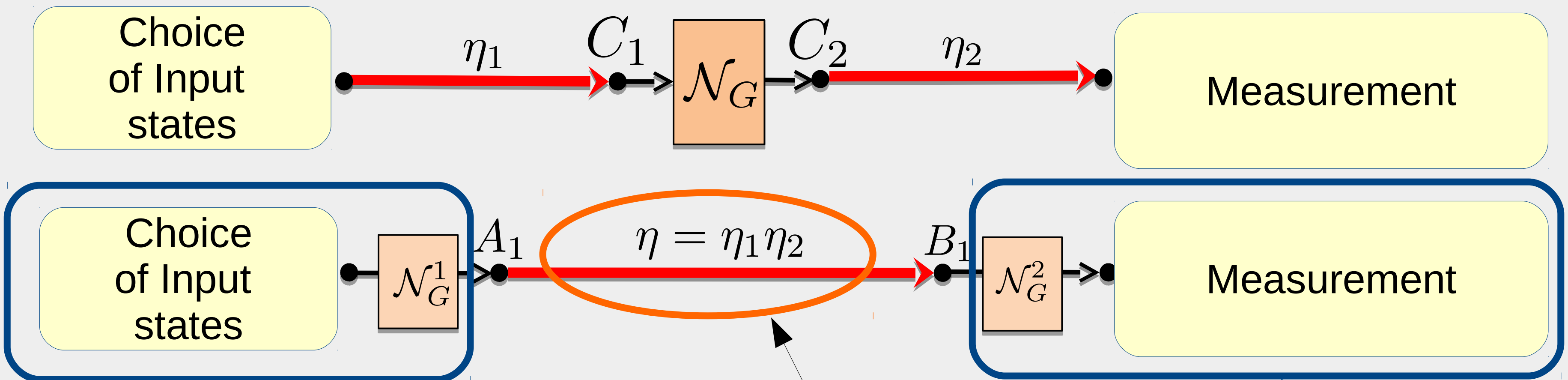


Another protocol with the original lossy channel!

Input source

Receiver settings

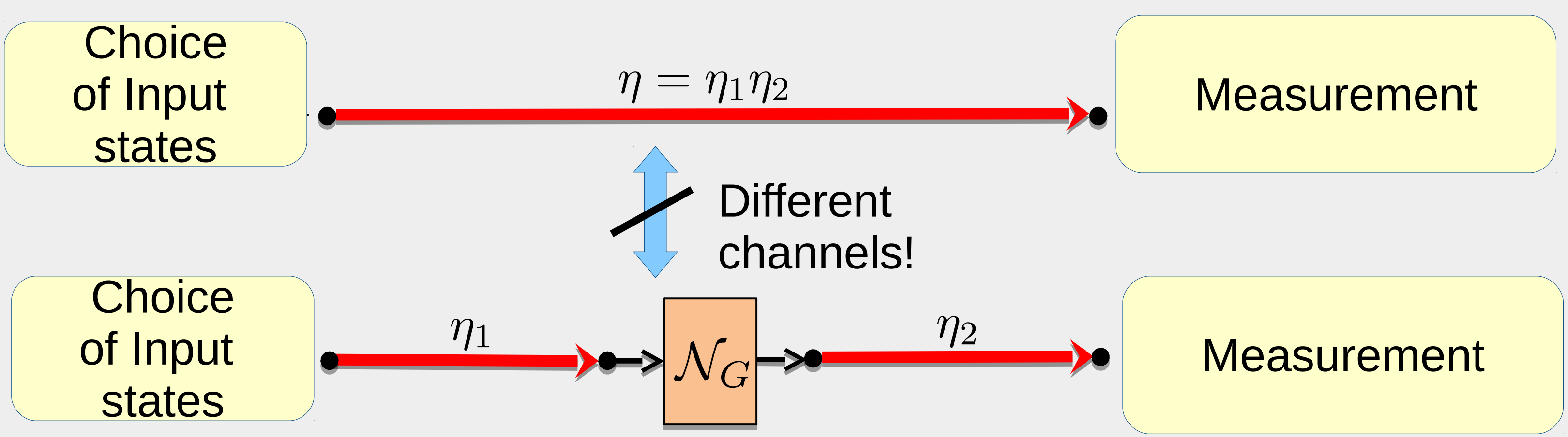
General bound of the secure key rate

$$R \leq \log_2\left(\frac{1+\eta}{1-\eta}\right) \sim 2.88\,\eta$$

can be applied.
[Takeoka, Guha, Wilde, arXiv:1310.0129]

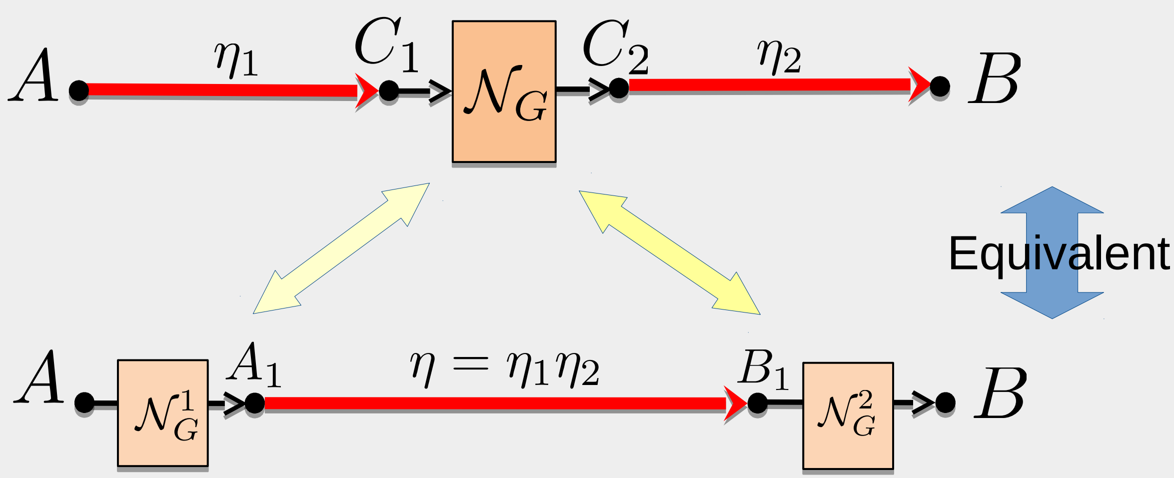
Remarks

- The performance may or may not improve
  if the transmitter and receiver are the same.



Remarks

- The performance may or may not improve
  if the transmitter and receiver are the same.
- Mathematically equivalent, not technically or economically.



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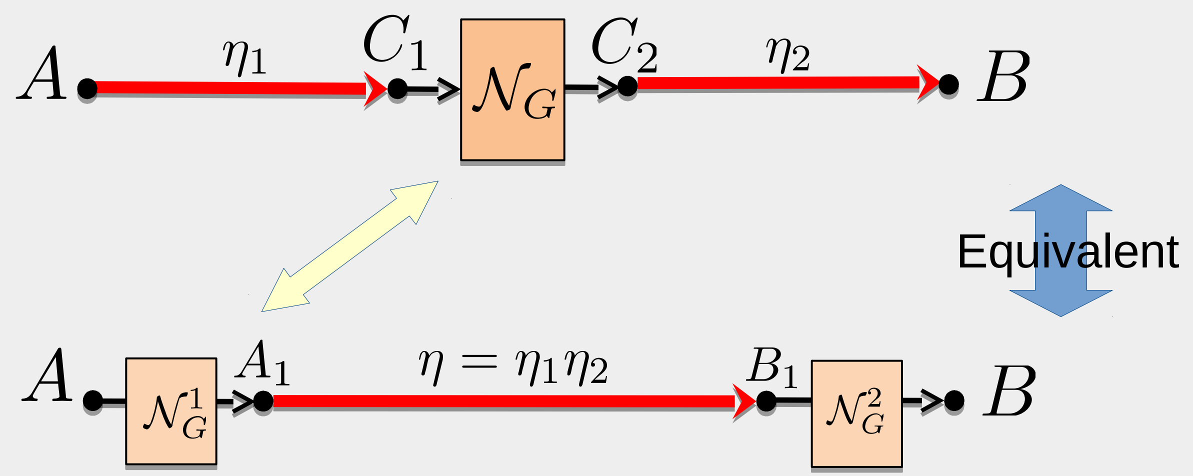
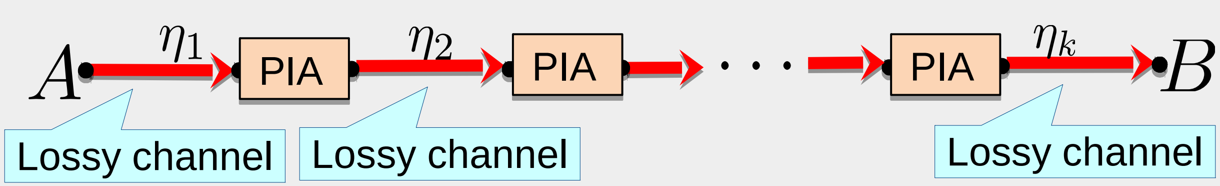


-Classical Communication

Phase insensitive amplifier (PIA) extends transmission distance



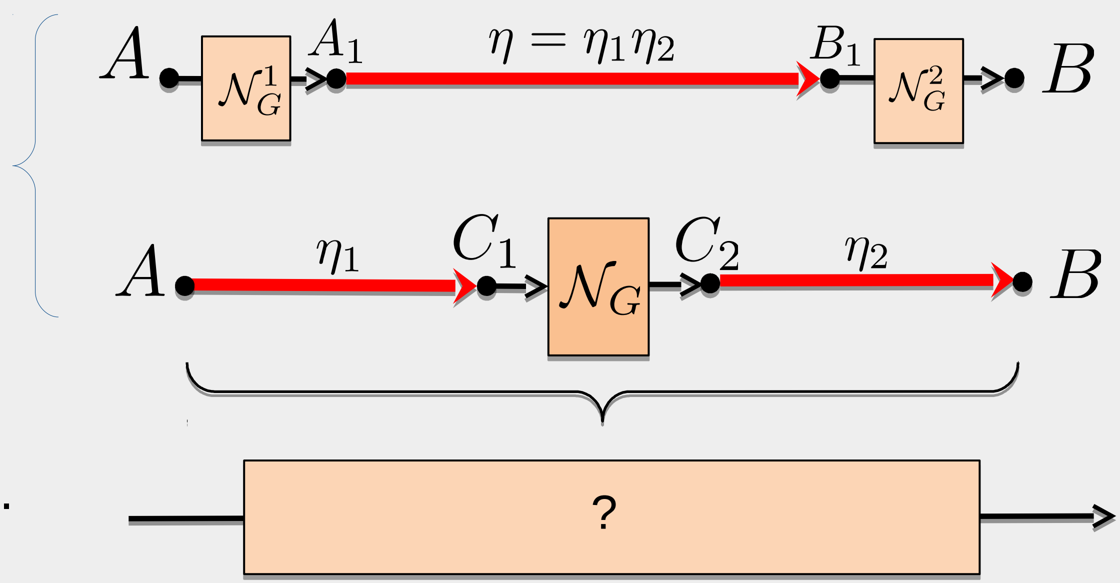
Single center station between lossy channels

- How does a center station modify the total channel?

-Single-mode Gaussian channels
-Entanglement breaking (EB) conditions

Equivalent decomposition

As a single module....



Single center station between lossy channels

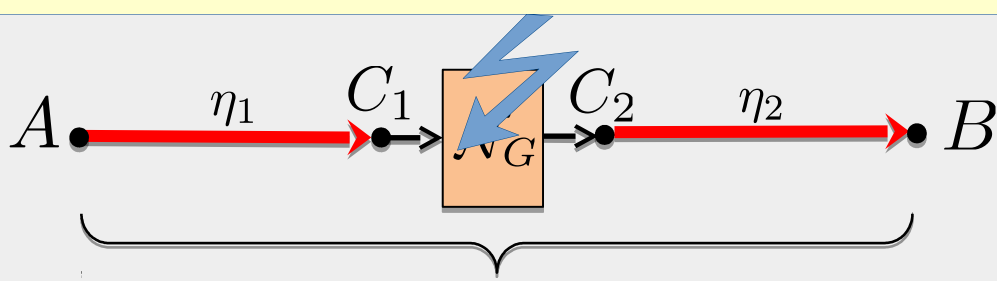
- How does a center station modify the total channel?

  -Single-mode Gaussian channels
  -Entanglement breaking (EB) conditions

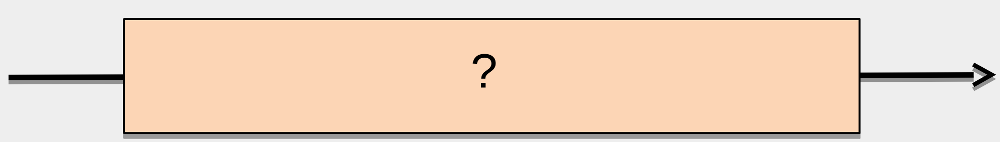
<u>Break any quantum correlation</u>

No entanglement / No secret key

Curty, Lewenstein & Lütkenhaus, Phys. Rev. Lett. 92, 217903 (2004).



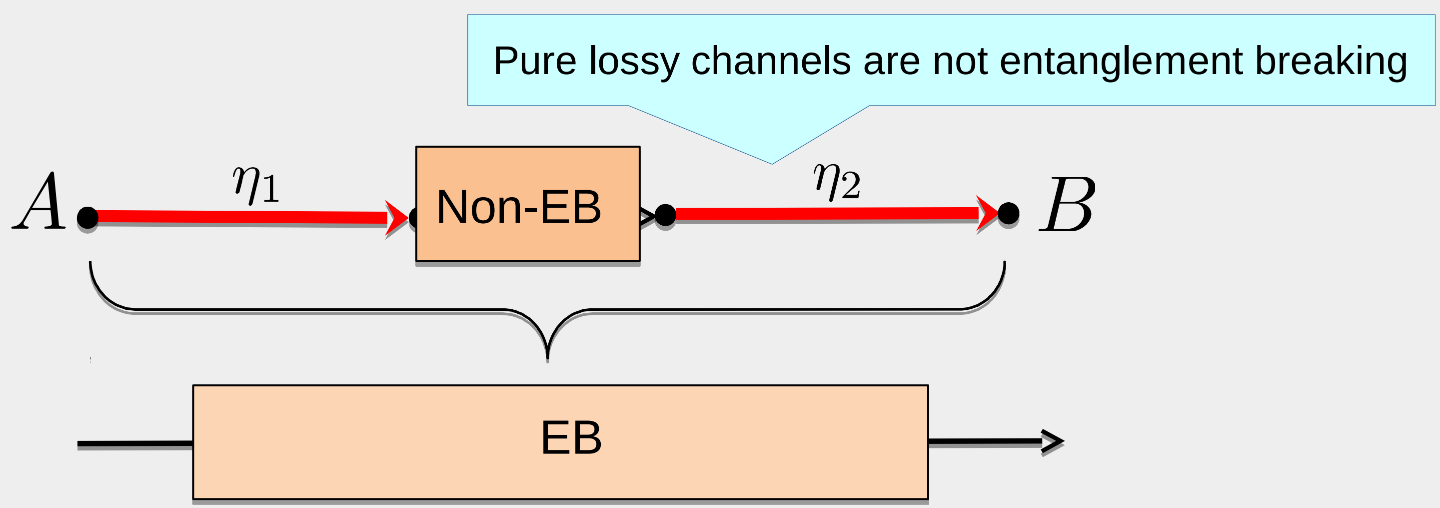
As a single module....



Single center station between lossy channels

- How does a center station modify the total channel?

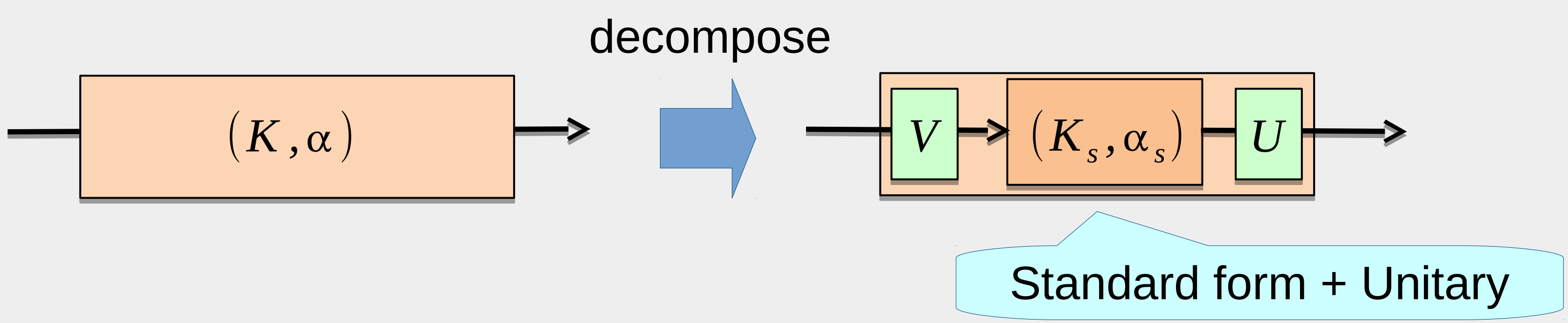
  -Single-mode Gaussian channels
  -Entanglement breaking (EB) conditions
  EB center station yields total EB channel.
  All we have to concern about is non-EB center stations!



As a single module....

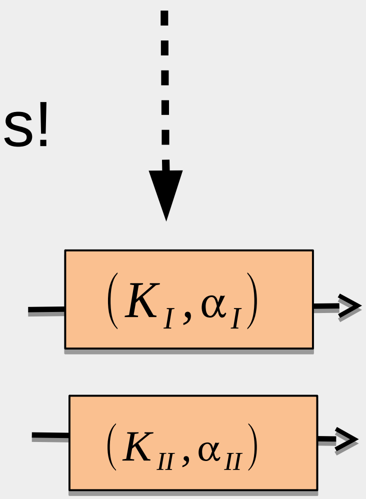
Single-mode Gaussian channels

- Unitary equivalent classification



Sufficient to consider two standard forms!

(I) Phase insensitive channel (PIC)
(II) Additive noise channel (ANC)



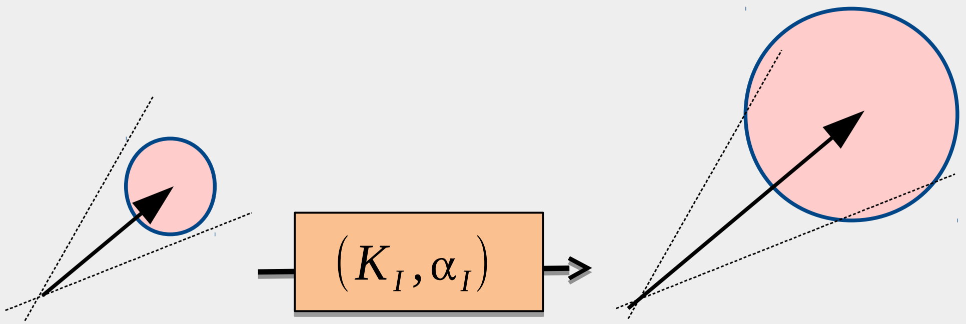
Holevo, Probl. Inf. Trans., 43, 1 (2007); Probl. Inf. Trans., 44, 171 (2008)

Non-entanglement breaking center stations

(I) Phase insensitive channel (PIC)

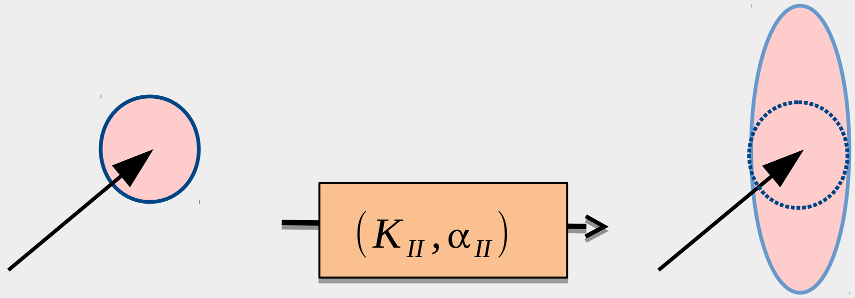
– Phase insensitive amplification/attenuation

Phase insensitive noise addition



(II) Additive noise channel (ANC)

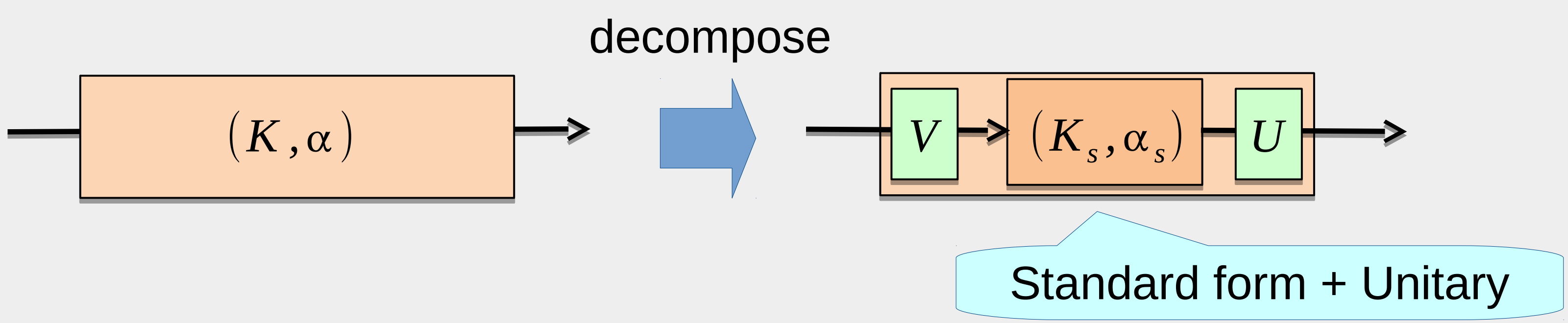
– Addition of a rank-1 noise



Holevo, Probl. Inf. Trans., 43, 1 (2007); Probl. Inf. Trans., 44, 171 (2008)

Single-mode Gaussian channels

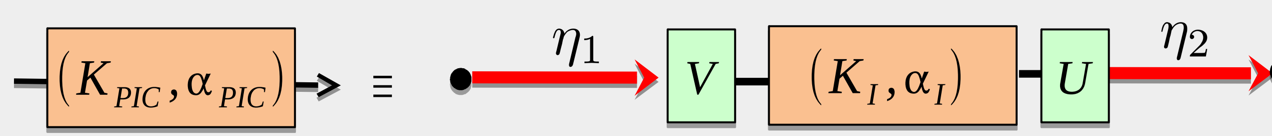
- Unitary equivalent classification



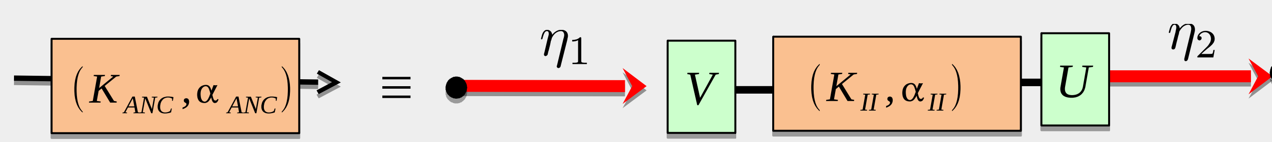
Sufficient to consider two standard forms!

- Entanglement breaking (EB) conditions:

$$\sqrt{det(\alpha_{PIC})} \geq \frac{1}{2}(1+g\,\eta_1\eta_2)$$



$$\sqrt{det(\alpha_{ANC})} \geq \frac{1}{2}(1+\eta_1\eta_2)$$



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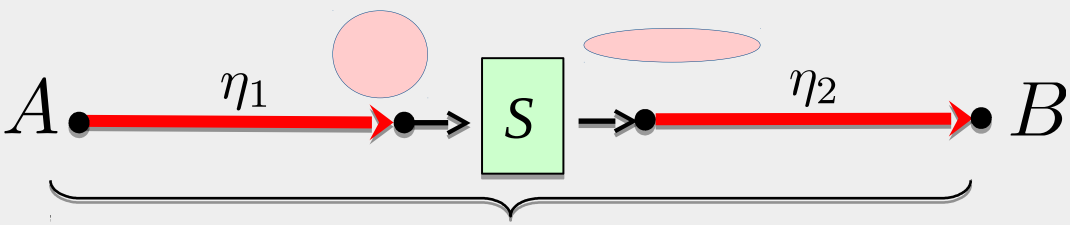
Examples: Quantum limited amplifiers

- Squeezer (Phase sensitive amplifier:PSA)



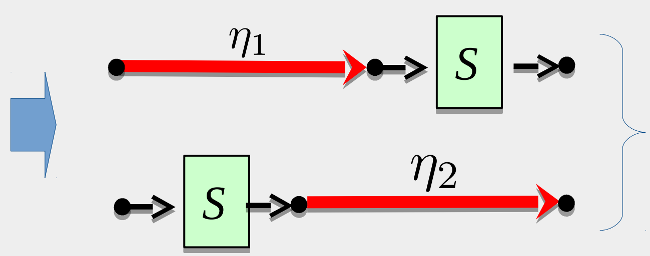
$$\frac{\Delta p^2}{\Delta x^2} = \left(\frac{\sqrt{G} - \sqrt{G-1}}{\sqrt{G} + \sqrt{G-1}} \right)^2, \; G \geq 1$$

$$K = Diag\left[\sqrt{G} + \sqrt{G-1}, \sqrt{G} - \sqrt{G-1}\right]$$

$$\alpha = 0$$

$\eta_2 = 1$

$\eta_1 = 1$



Unitary equivalent to a pure lossy channel

⇒ Not EB!

Examples: Quantum limited amplifiers

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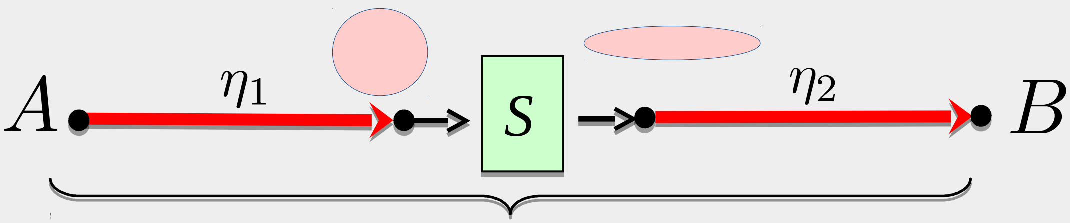

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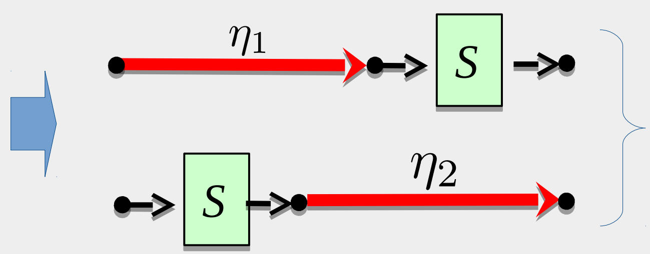
$$K = Diag\left[\sqrt{G} + \sqrt{G-1}, \sqrt{G} - \sqrt{G-1}\right]$$

$$\alpha = 0$$

EB if the gain fulfils $$G_{\mathrm{PSA}} \geq 1 + \frac{\eta_1}{(1-\eta_1)(1-\eta_2)}$$

$\eta_2 = 1$

$\eta_1 = 1$



Unitary equivalent to a pure lossy channel
$\Rightarrow$ Not EB!

A middle unitary operation renders the channel entanglement breaking!

Examples: Quantum limited amplifiers

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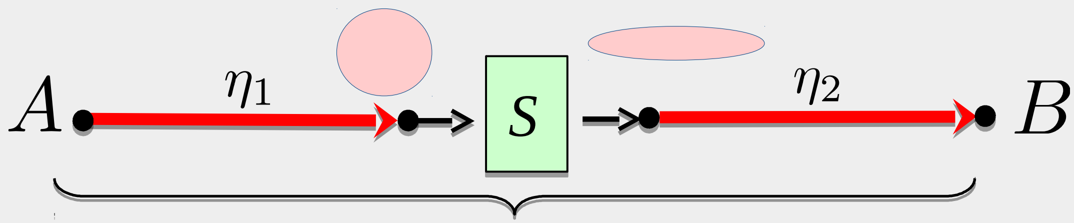
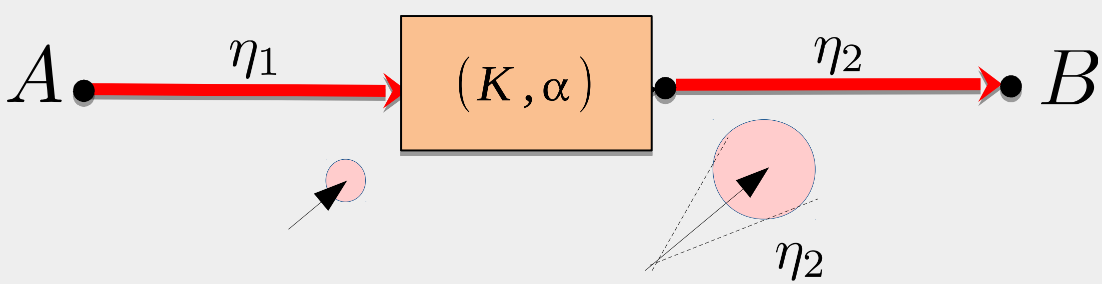


$$\frac{\Delta p^2}{\Delta x^2}=\left(\frac{\sqrt{G}-\sqrt{G-1}}{\sqrt{G}+\sqrt{G-1}}\right)^2,\ G\geq 1$$

$$K=Diag\left[\sqrt{G}+\sqrt{G-1},\sqrt{G}-\sqrt{G-1}\right]$$
$$\alpha=0$$

EB if the gain fulfils
$$G_{\mathrm{PSA}}\geq 1+\frac{\eta_1}{(1-\eta_1)(1-\eta_2)}$$

- Quantum-limited phase insensitive amplifier (PIA)

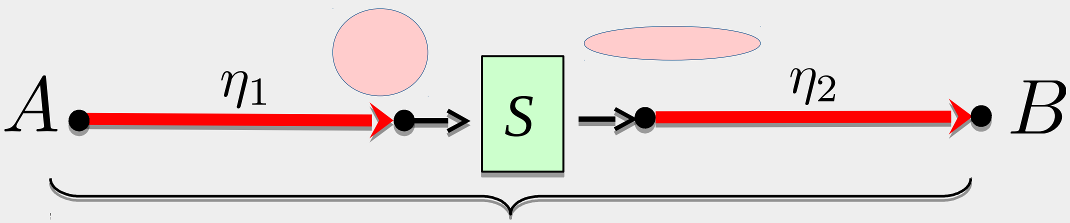


$$\frac{X_{output}}{X_{input}}=\sqrt{G}$$

$$K=\sqrt{G}\,I_2$$
$$\alpha=|1-G|\,I_2/2$$

Examples: Quantum limited amplifiers

- Squeezer (Phase sensitive amplifier:PSA)

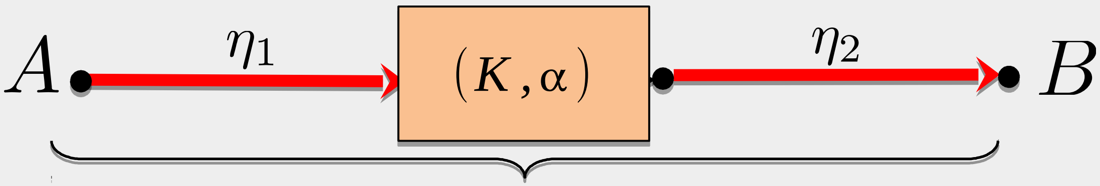


$$\frac{\Delta p^2}{\Delta x^2} = \left(\frac{\sqrt{G}-\sqrt{G-1}}{\sqrt{G}+\sqrt{G-1}}\right)^2, \; G \geq 1$$

$$K = Diag[\sqrt{G}+\sqrt{G-1}, \sqrt{G}-\sqrt{G-1}]$$
$$\alpha = 0$$

EB if the gain fulfils $$G_{\mathrm{PSA}} \geq 1 + \frac{\eta_1}{(1-\eta_1)(1-\eta_2)}$$

- Quantum-limited phase insensitive amplifier (PIA)



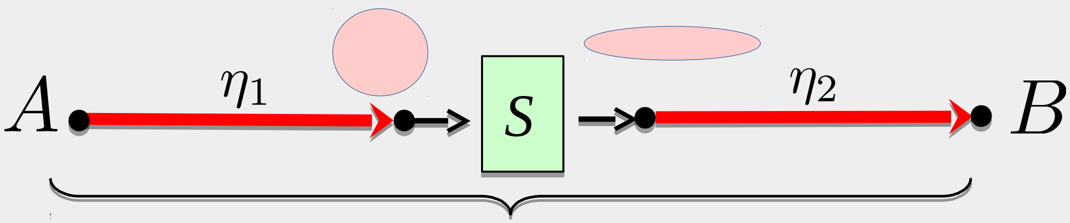
$$\frac{X_{output}}{X_{input}} = \sqrt{G}$$

$$K = \sqrt{G}\, I_2$$
$$\alpha = |1-G|\, I_2/2$$

EB if the gain fulfils $$G_{\mathrm{PIA}} \geq \frac{1}{1-\eta_1}$$

Examples: Quantum limited amplifiers

- Squeezer (Phase sensitive amplifier:PSA)



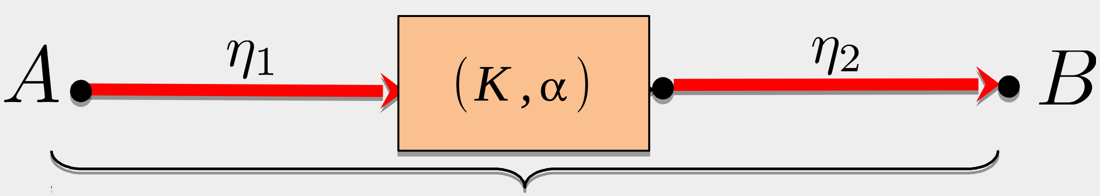
$$\frac{\Delta p^2}{\Delta x^2} = \left(\frac{\sqrt{G} - \sqrt{G-1}}{\sqrt{G} + \sqrt{G-1}} \right)^2, \; G \geq 1$$

$$K = Diag\left[\sqrt{G} + \sqrt{G-1}, \sqrt{G} - \sqrt{G-1}\right]$$

$$\alpha = 0$$

EB if the gain fulfils $G_{\mathrm{PSA}} \geq 1 + \frac{\eta_1}{(1-\eta_1)(1-\eta_2)}$

- Quantum-limited phase insensitive amplifier (PIA)



$$\frac{X_{output}}{X_{input}} = \sqrt{G}$$

$$K = \sqrt{G} I_2$$

$$\alpha = |1-G| I_2 / 2$$

EB if the gain fulfils $G_{\mathrm{PIA}} \geq \frac{1}{1-\eta_1}$

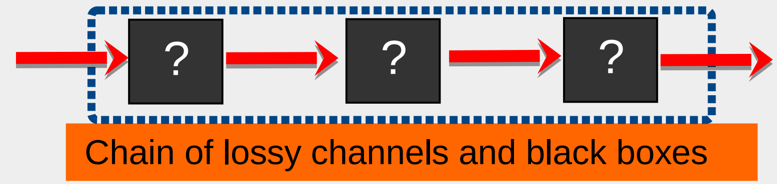
For a long distance $\eta_1, \eta_2 \ll 1 \rightarrow G \gtrsim 1$

– a small gain amplification renders the channel entanglement breaking!

Conclusion

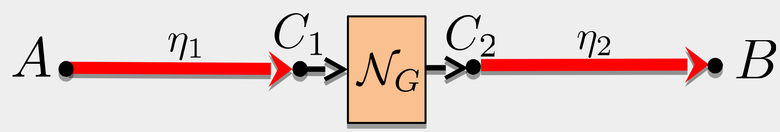
- Useful black boxes do exist!

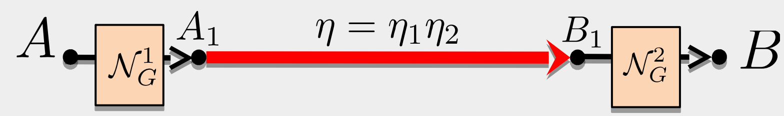
  Break the linear scaling R ~ η



- No-go result for Gaussian center stations

  General multi-mode Gaussian channels

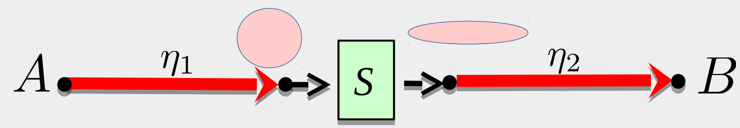




- Conditions that a single-mode Gaussian station make whole channel entanglement breaking (EB)

  Special cases: quantum limited amplifier

  – A small amplification could make the channel entanglement breaking Better off using amplifiers as repeater stations



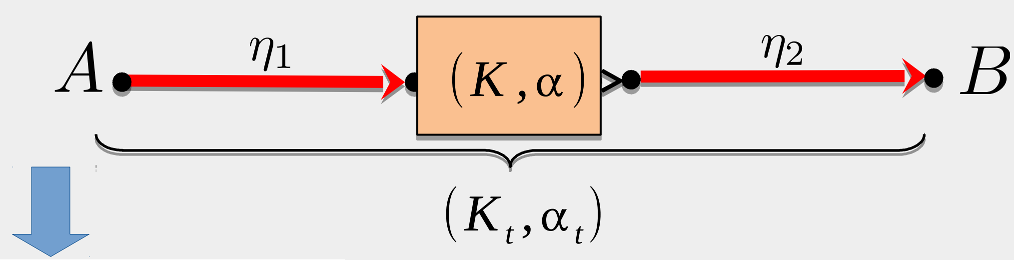
Construction

Sketch of Proof

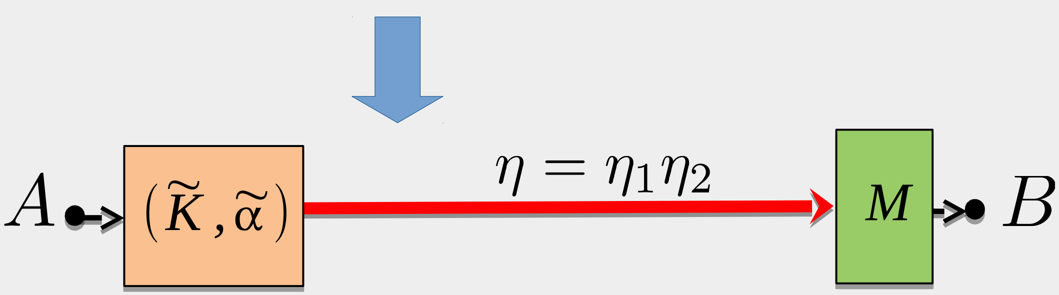
I. The total channel action

II. Existence of a noise term and a unitary matrix

III. Choose a gain term



$$\gamma' := \frac{\alpha_t - \eta_1 \eta_2 \alpha}{1 - \eta_1 \eta_2}, \quad M^T \gamma' M \geq \frac{1}{2} I_{2n}$$



$$\tilde{K} = \frac{1}{\sqrt{\eta_1 \eta_2}} K M$$

$$\tilde{\alpha} := \frac{1}{\eta_1 \eta_2} \left(M^T \alpha_t M - \frac{1 - \eta_1 \eta_2}{2} I_{2n} \right)$$