

On the inefficacy of Gaussian regenerative amplifiers for quantum optical communication

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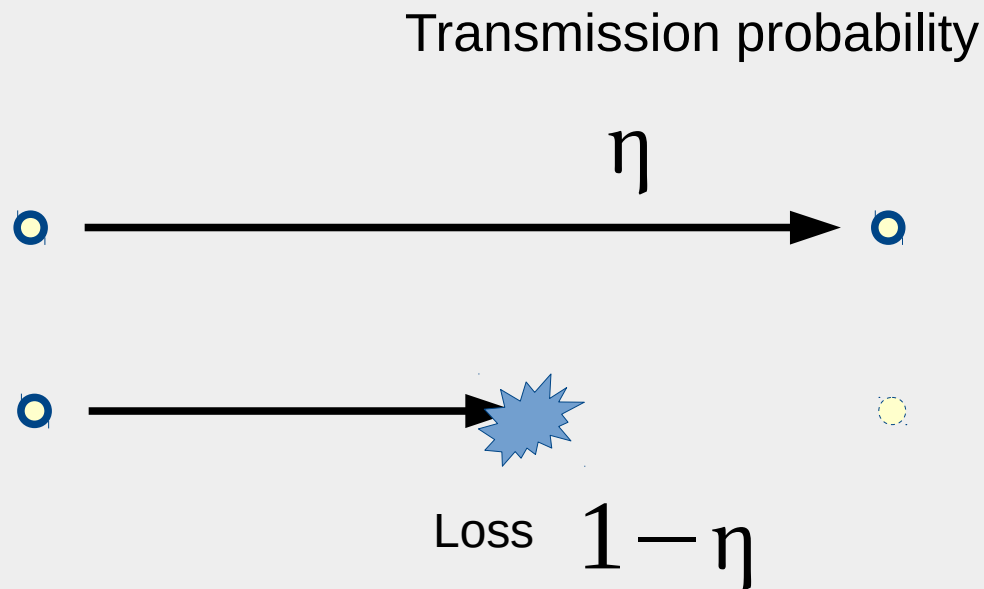
Quantum Information Processing group, BBN Technologies



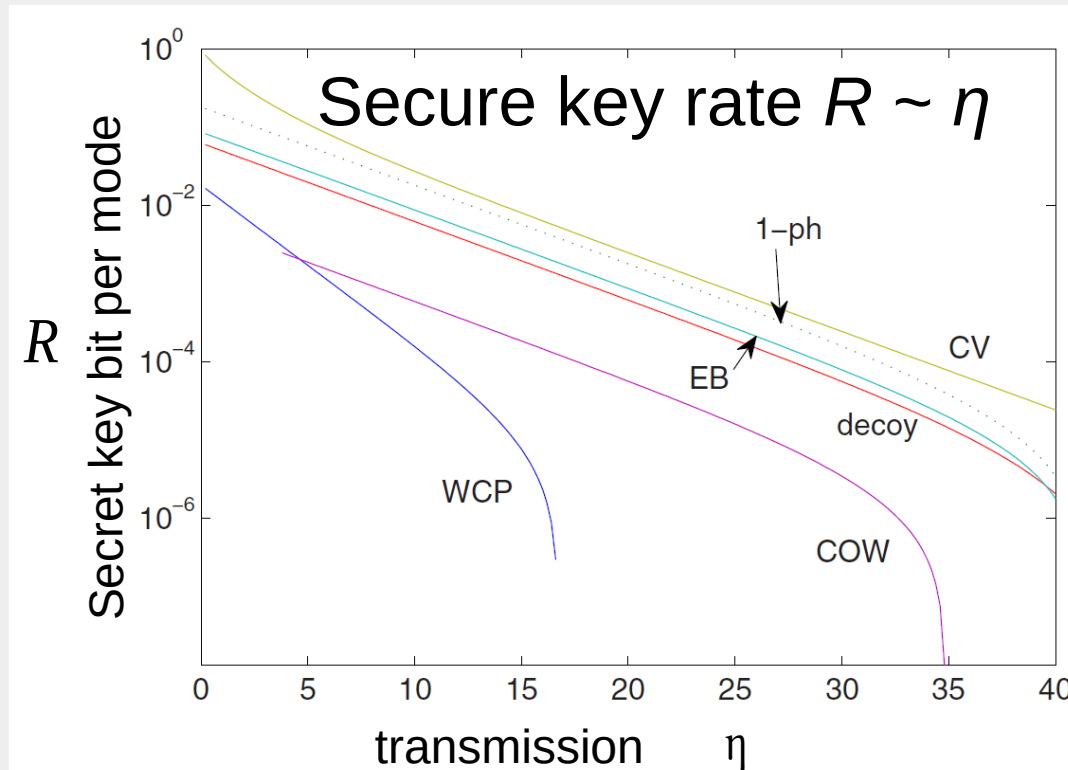
DARPA Quiness Program
Office of Naval Research

Introduction

- The photon loss limits long distance QKD.



Introduction

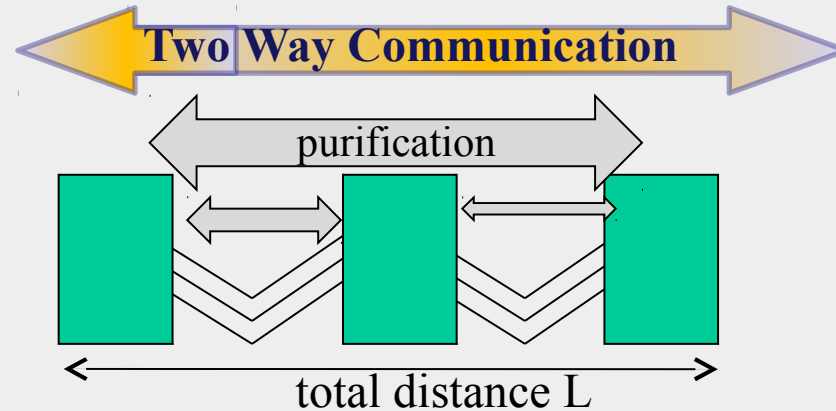


— Scarani et al., Rev. Mod. Phys. 81, 1301 (2009)

- General upper bound on the secure key rate $R \leq \log_2\left(\frac{1+\eta}{1-\eta}\right) \sim 2.88\eta$
- Exponential decay with distance $\eta = e^{-L_{tot}/L_0}$ ($\eta \ll 1$)

Introduction

- Quantum Repeater
1st generation

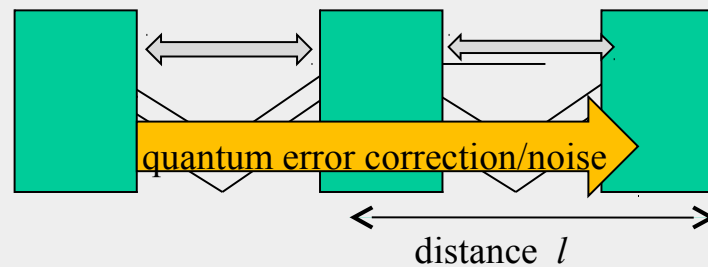


Classical communication

Repetition rate

$$R < \frac{c}{L}$$

- 2nd generation

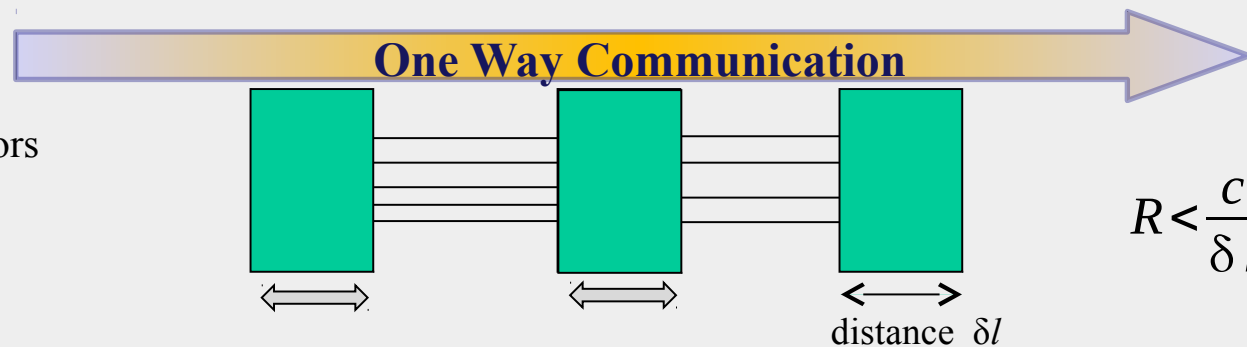


$$R < \frac{c}{l}$$

- 3rd generation

QECC for operational errors

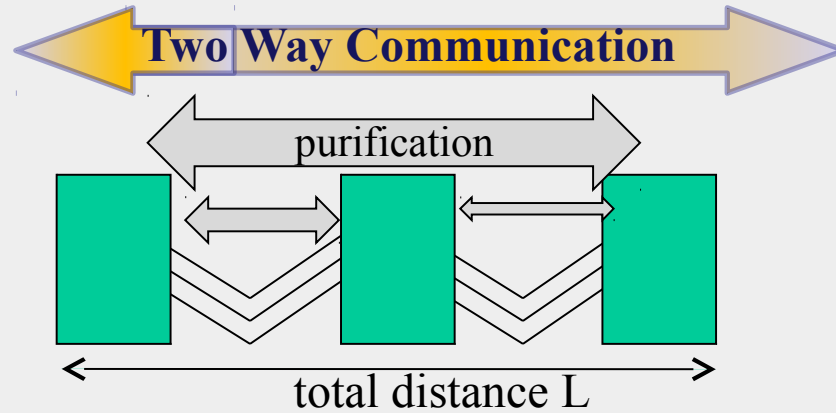
QECC for loss



$$R < \frac{c}{\delta l}$$

Introduction

- Quantum Repeater 1st generation

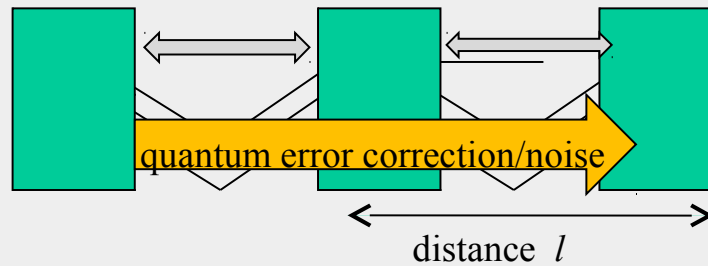


Classical communication

Repetition rate

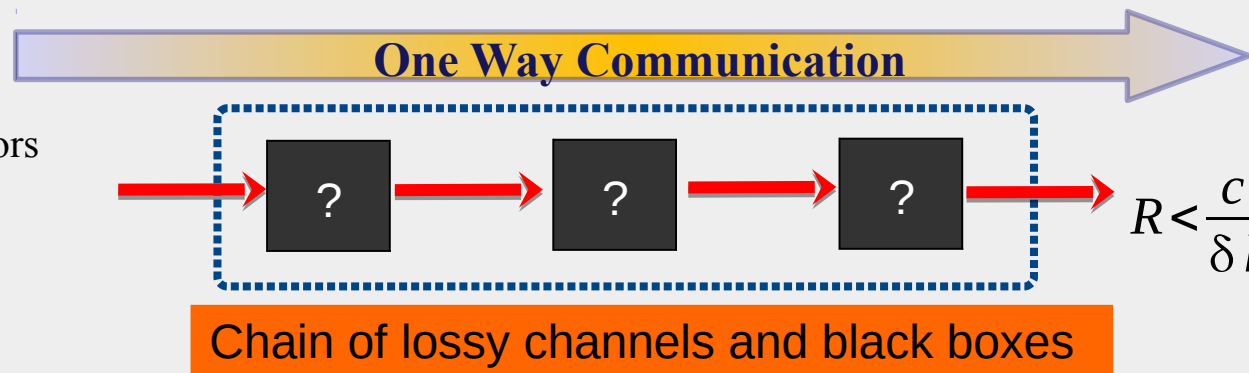
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$$R < \frac{c}{l}$$

- 3rd generation



QECC for operational errors

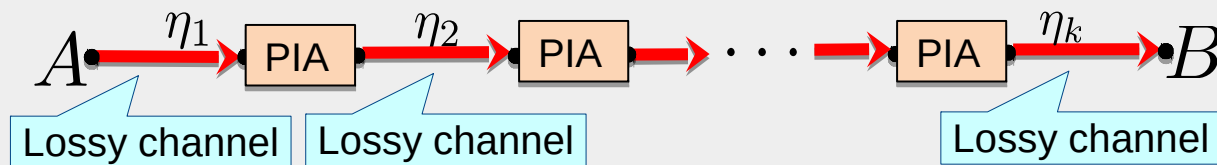
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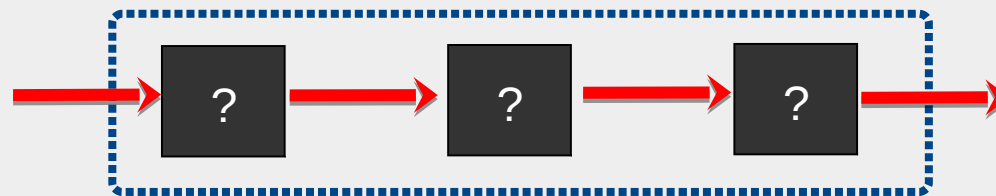
Introduction

- Classical Communication

Phase insensitive amplifier (PIA) extends transmission distance



- Quantum Communication:
3rd generation quantum repeater

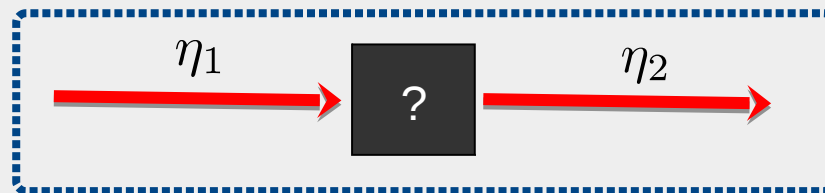


Chain of lossy channels and black boxes

What could those black boxes be?

Single center station between lossy channel

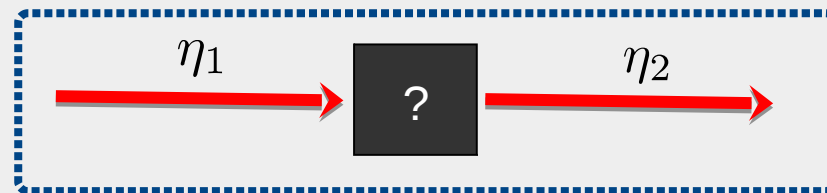
- How does a center station modify the total channel?
- Simplest tool box: Gaussian operations
- Could **phase insensitive amplifiers (PIA)** or generally **Gaussian quantum channels** work as a quantum repeater?



Chain of lossy channels and black boxes

Single center station between lossy channel

- How does a center station modify the total channel?
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Chain of lossy channels and black boxes

Gaussian states

Quantum states of Light: harmonic oscillators

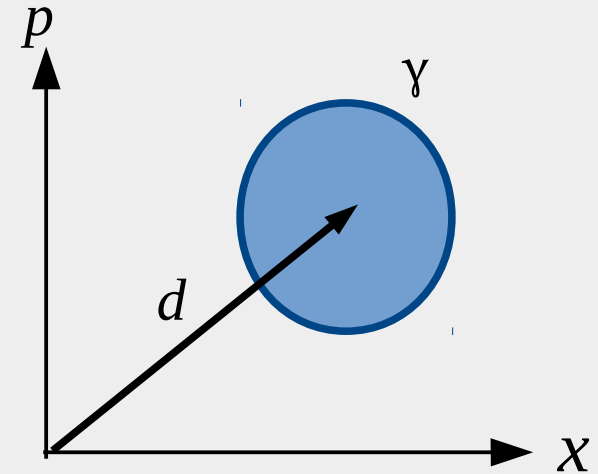
n -mode bosonic field

- Displacement vector

$$d = \begin{bmatrix} \square \\ \square \\ \square \\ \square \end{bmatrix} \left. \vphantom{\begin{bmatrix} \square \\ \square \\ \square \\ \square \end{bmatrix}} \right\} 2n$$

- Covariance matrix

$$\gamma = \begin{bmatrix} \square & \square & \square & \square \\ \square & \square & \square & \square \\ \square & \square & \square & \square \\ \square & \square & \square & \square \end{bmatrix} \quad 2n \times 2n$$



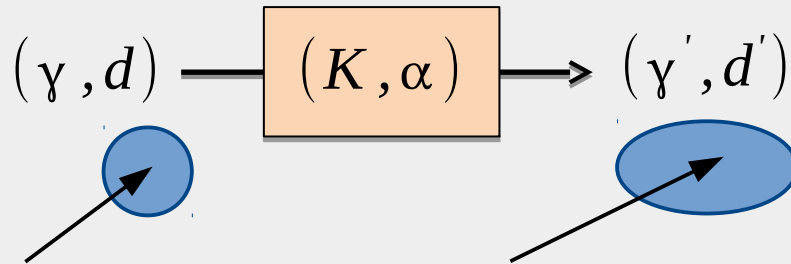
$$\Delta x \Delta p \geq 1/2$$

canonical uncertainty relation

$$\left\{ \begin{array}{l} \gamma \geq \frac{i}{2} \sigma, \quad \sigma := \begin{pmatrix} 0 & I_n \\ I_n & 0 \end{pmatrix} \\ \text{Identity matrix: } I_n \end{array} \right.$$

Gaussian channels

Def. Transform Gaussian states to Gaussian states



$(K, \alpha) = (\text{Gain, Noise})$

$$\begin{cases} d' = K^T d, \\ \gamma' = K^T \gamma K + \alpha \end{cases}$$

$$K = \begin{bmatrix} \square & \square & \square & \square \\ \square & \square & \square & \square \\ \square & \square & \square & \square \\ \square & \square & \square & \square \end{bmatrix}$$

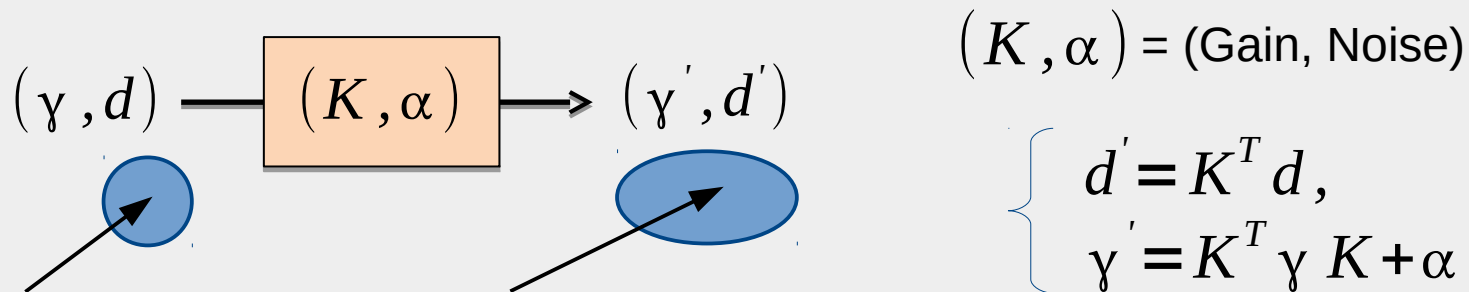
$$\alpha = \begin{bmatrix} \square & \square & \square & \square \\ \square & \square & \square & \square \\ \square & \square & \square & \square \\ \square & \square & \square & \square \end{bmatrix} \quad 2n \times 2n$$

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canonical uncertainty relation

Physical Condition for channels

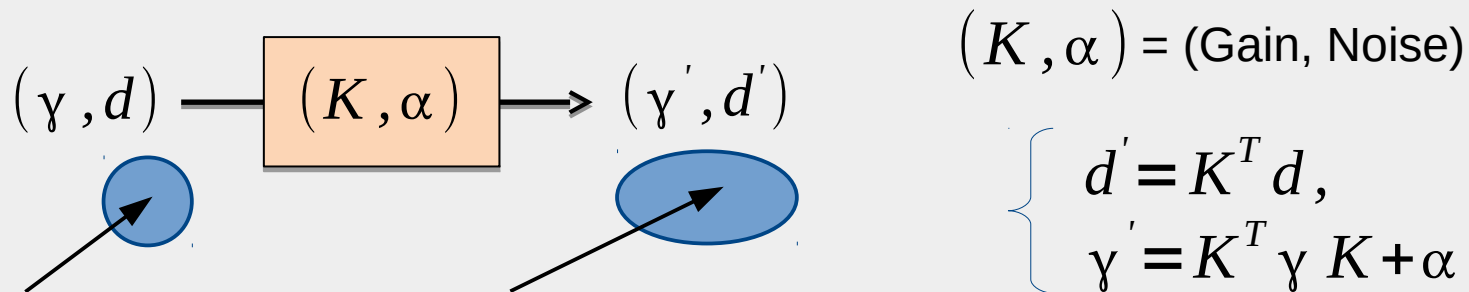
$$\alpha \geq \frac{i}{2} (\sigma - K^T \sigma K)$$

$$\gamma \geq \frac{i}{2} \sigma, \quad \sigma := \begin{pmatrix} 0 & I_n \\ I_n & 0 \end{pmatrix}$$

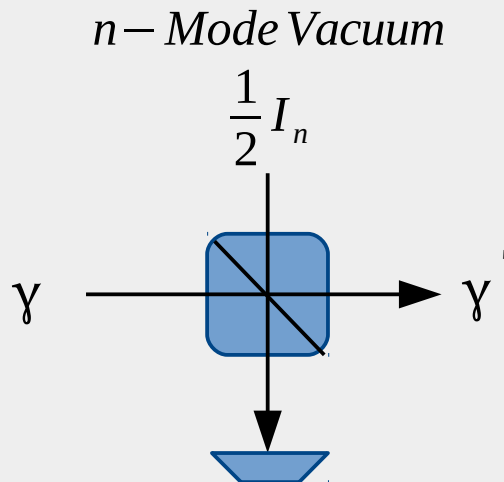
Identity matrix: I_n

Gaussian channels

Def. Transform Gaussian states to Gaussian states



- Multi-mode Pure lossy channels; transmission η

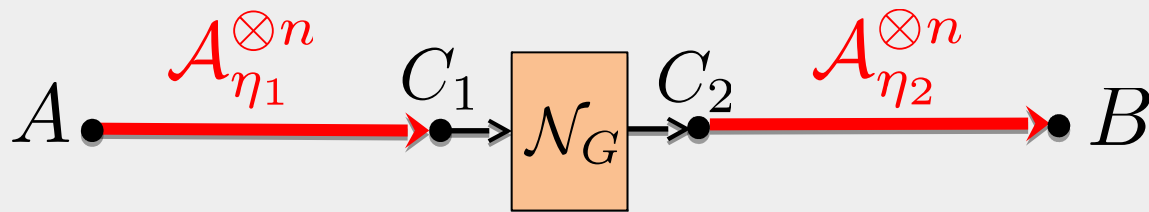


$$K = \sqrt{\eta} I_{2n}, \quad \alpha = \frac{1-\eta}{2} I_{2n}.$$

$$\begin{cases} d' = \sqrt{\eta} d, \\ \gamma' = \frac{1-\eta}{2} I_{2n} + \eta \gamma \end{cases}$$

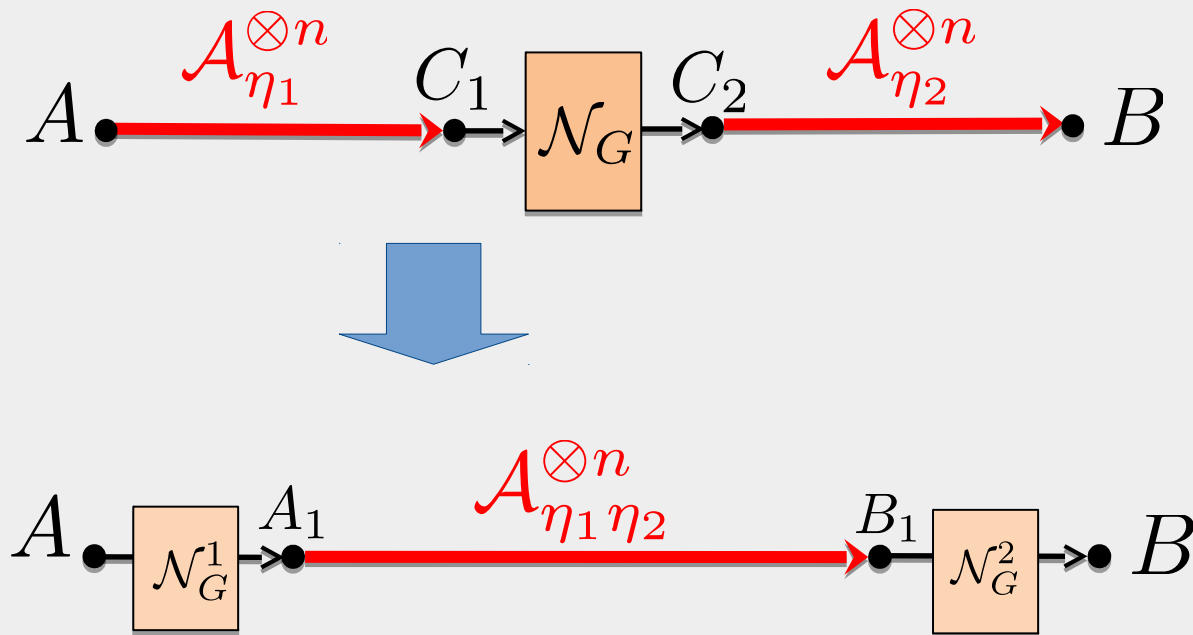
Setting

- Gaussian channel sandwiched between lossy channel segment



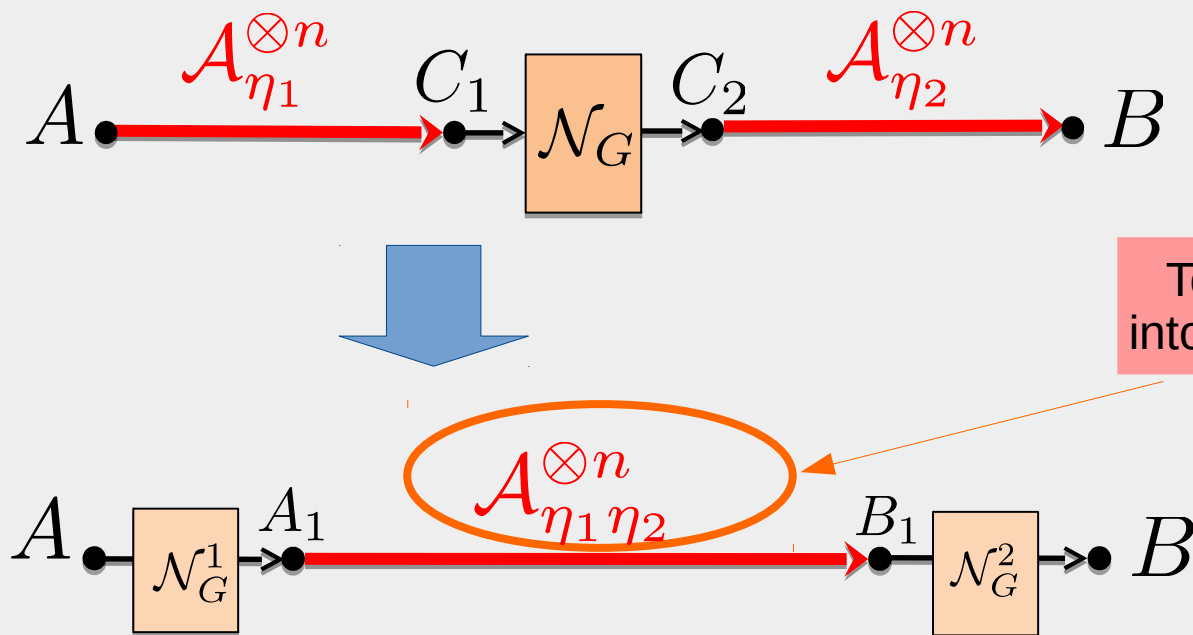
Main theorem

- Decomposition of Gaussian center station



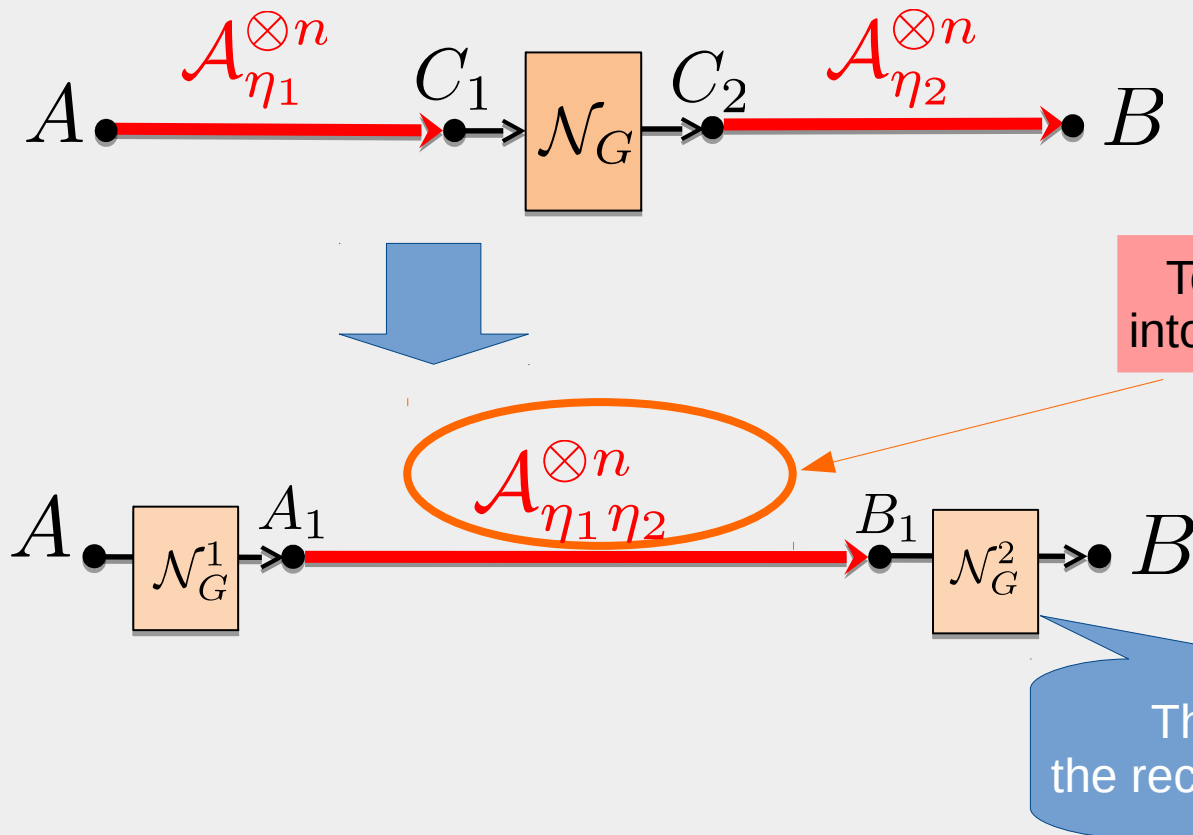
Main theorem

- Decomposition of Gaussian center station

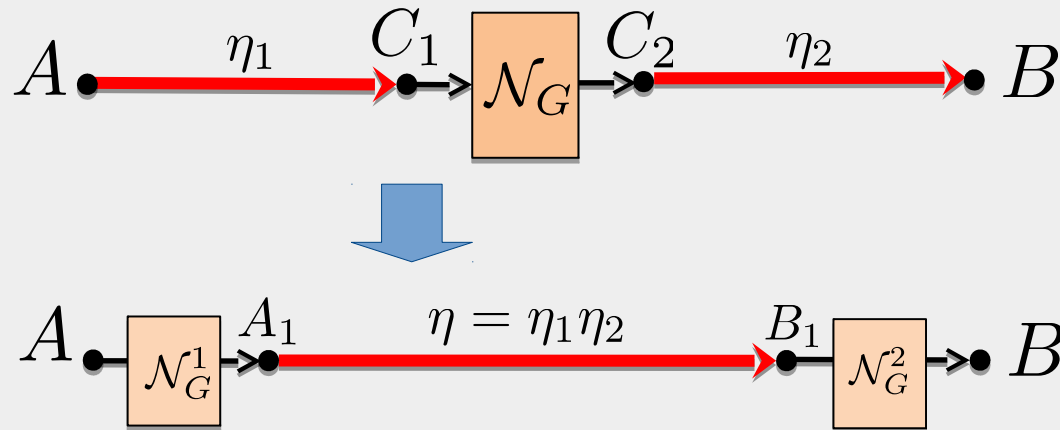


Main theorem

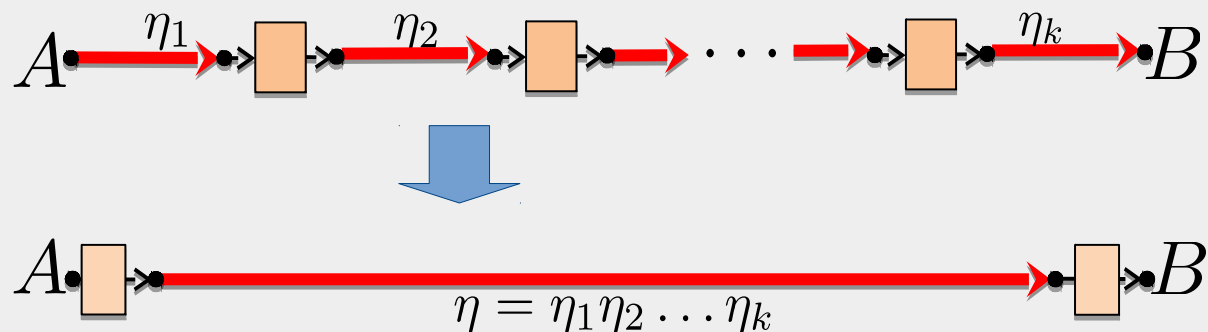
- Decomposition of Gaussian center station



Implication for many stations



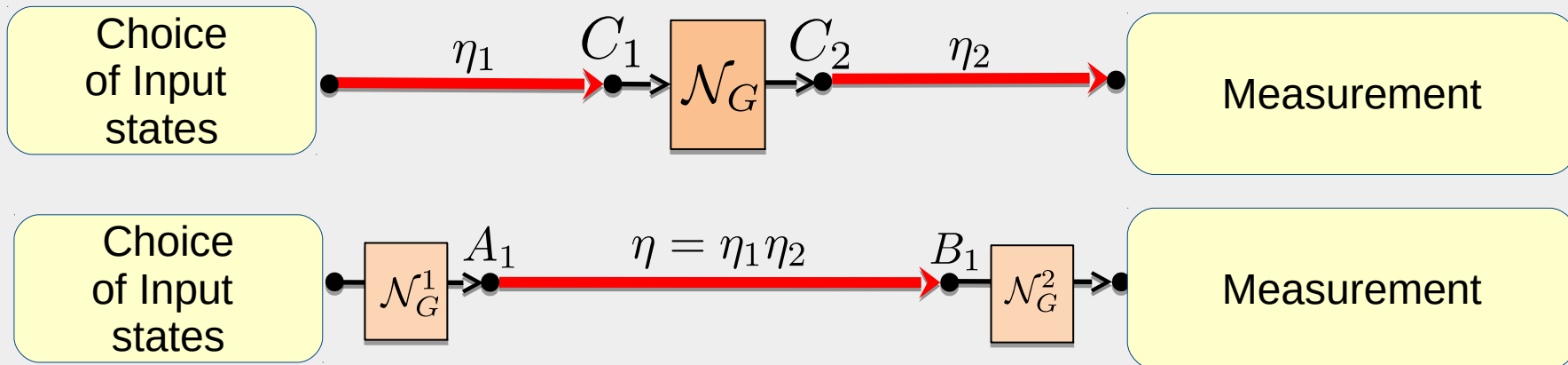
- Effect of loss cannot be reduced!
- Cannot be improved by Interspersing many stations!



★ No difference in Gaussian or Non-Gaussian input states

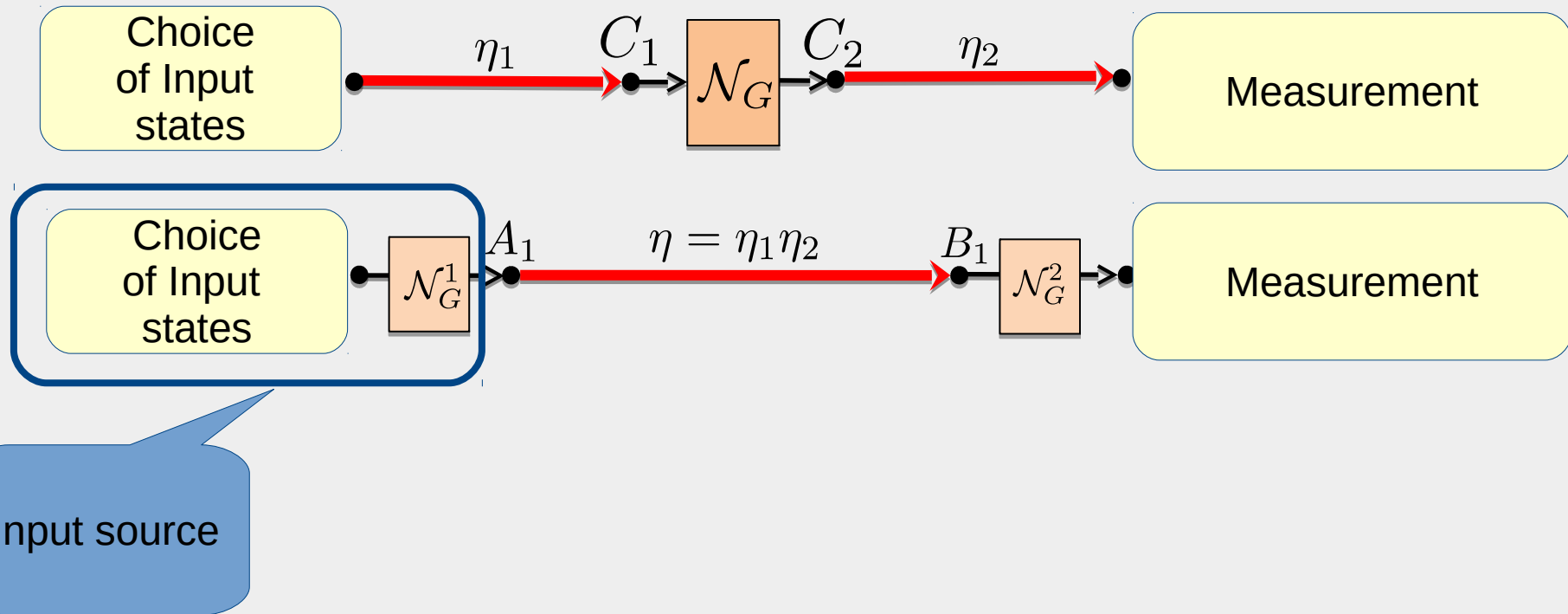
No-Go result for Gaussian repeater

- Center station \rightarrow Modification of transmitter and receiver.



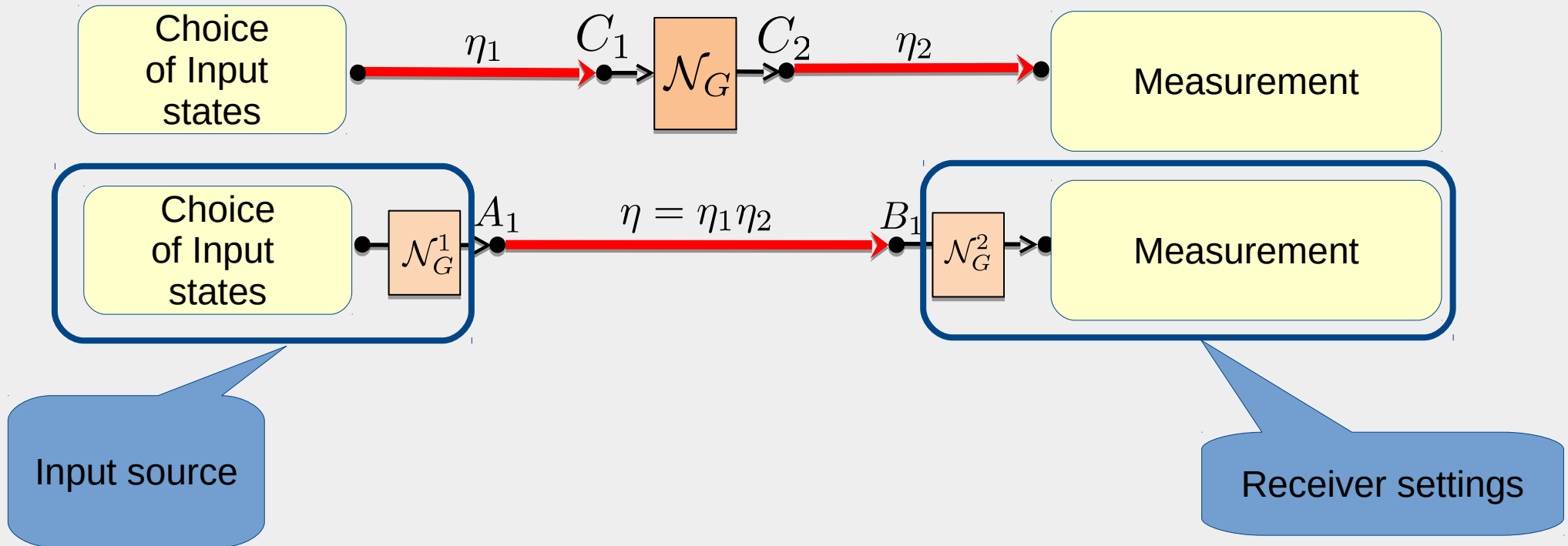
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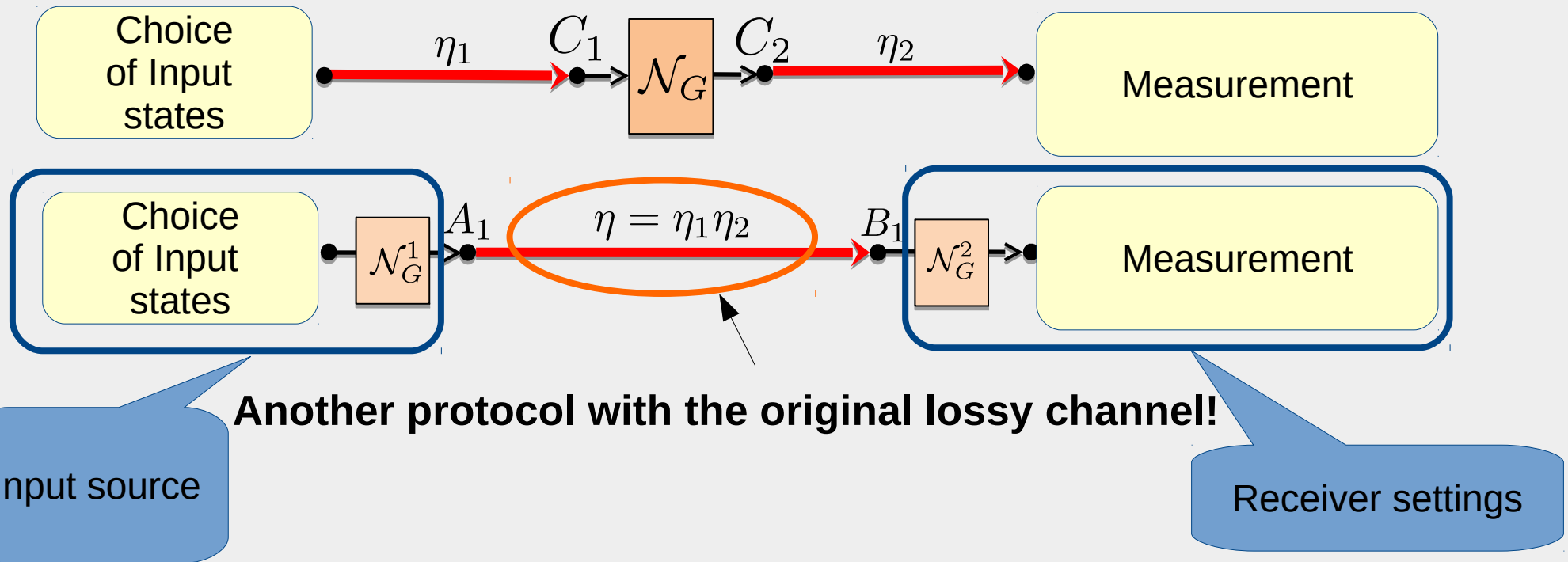
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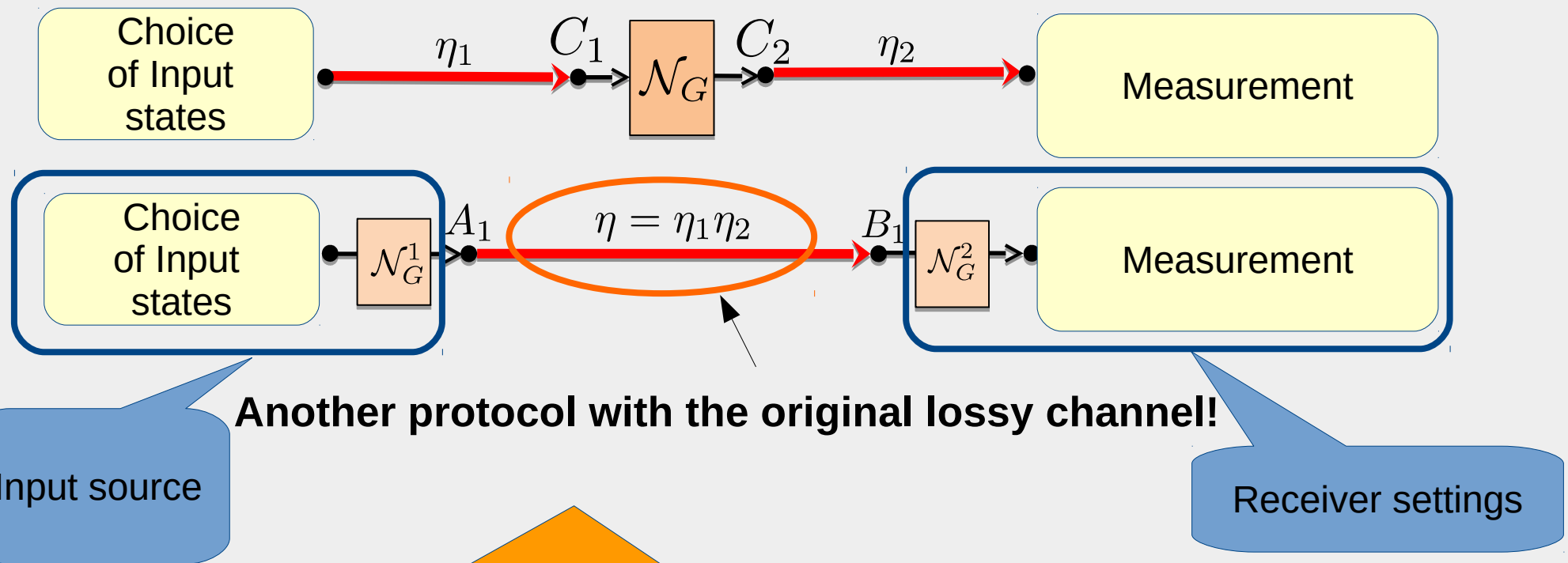
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No-Go result for Gaussian repeater

- Center station \rightarrow Modification of transmitter and receiver.



General bound of the secure key rate

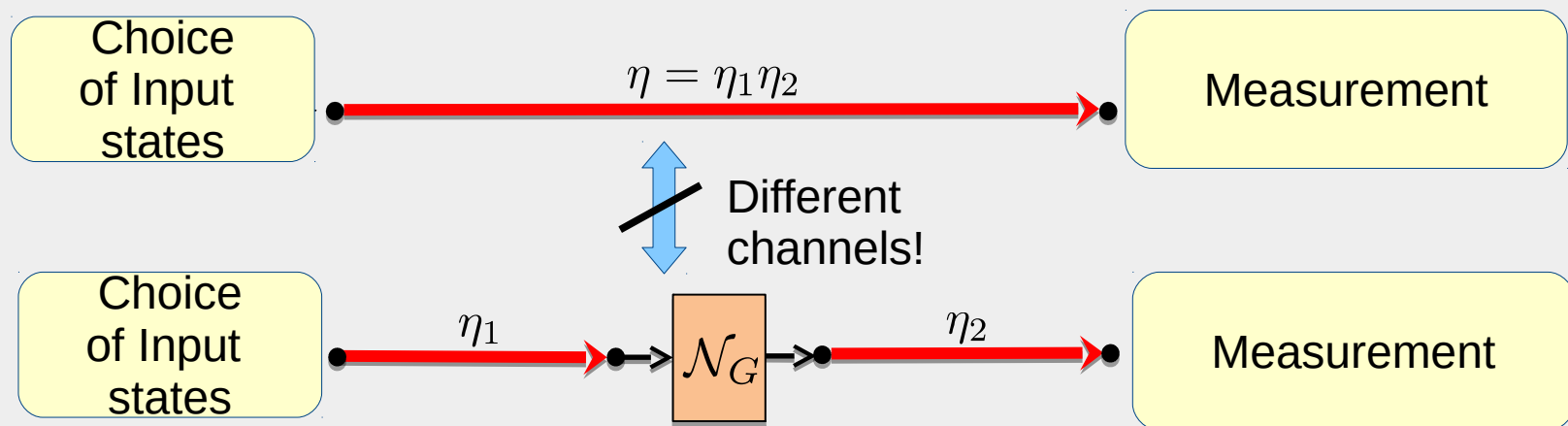
$$R \leq \log_2 \left(\frac{1+\eta}{1-\eta} \right) \sim 2.88 \eta$$

can be applied.

[Takeoka, Guha, Wilde, arXiv:1310.0129]

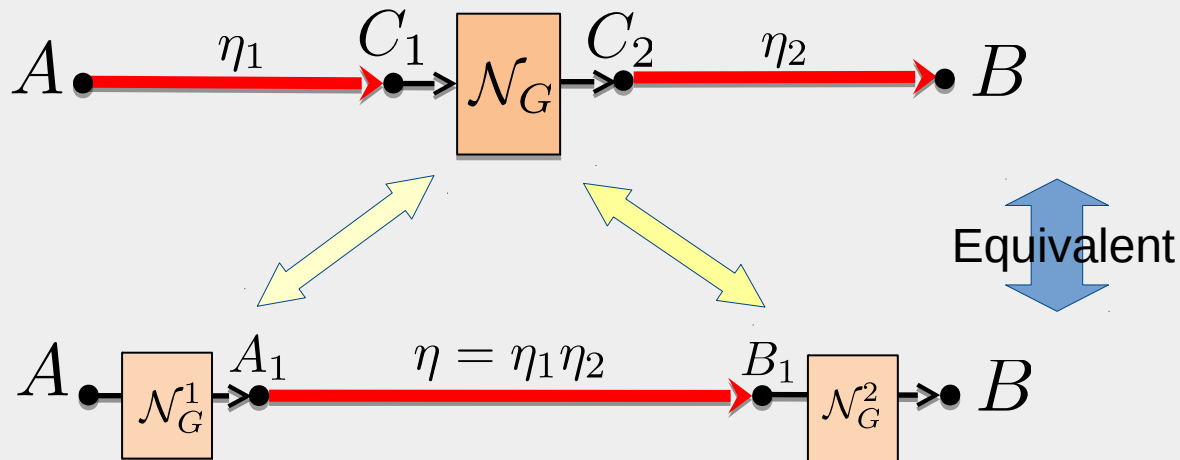
Remarks

- The performance may or may not improve if the transmitter and receiver are the same.



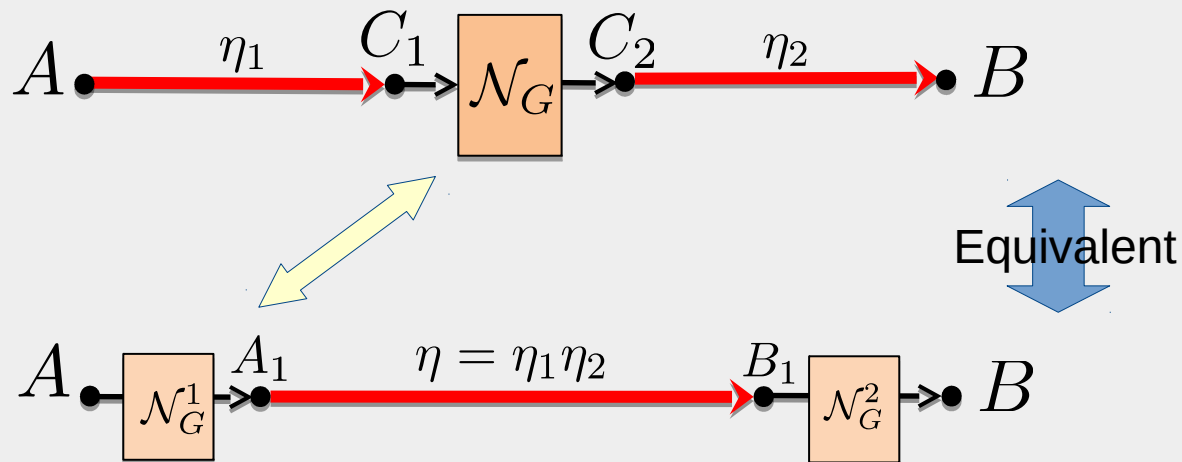
Remarks

- The performance may or may not improve if the transmitter and receiver are the same.
- Mathematically equivalent, not technically or economically.



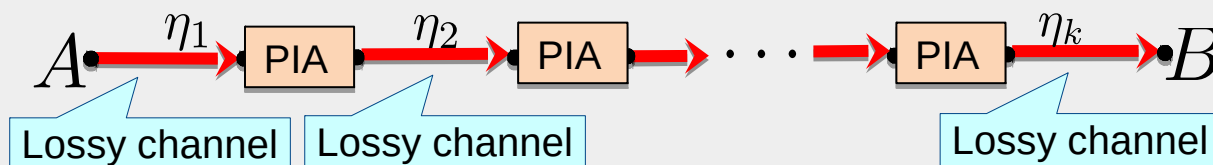
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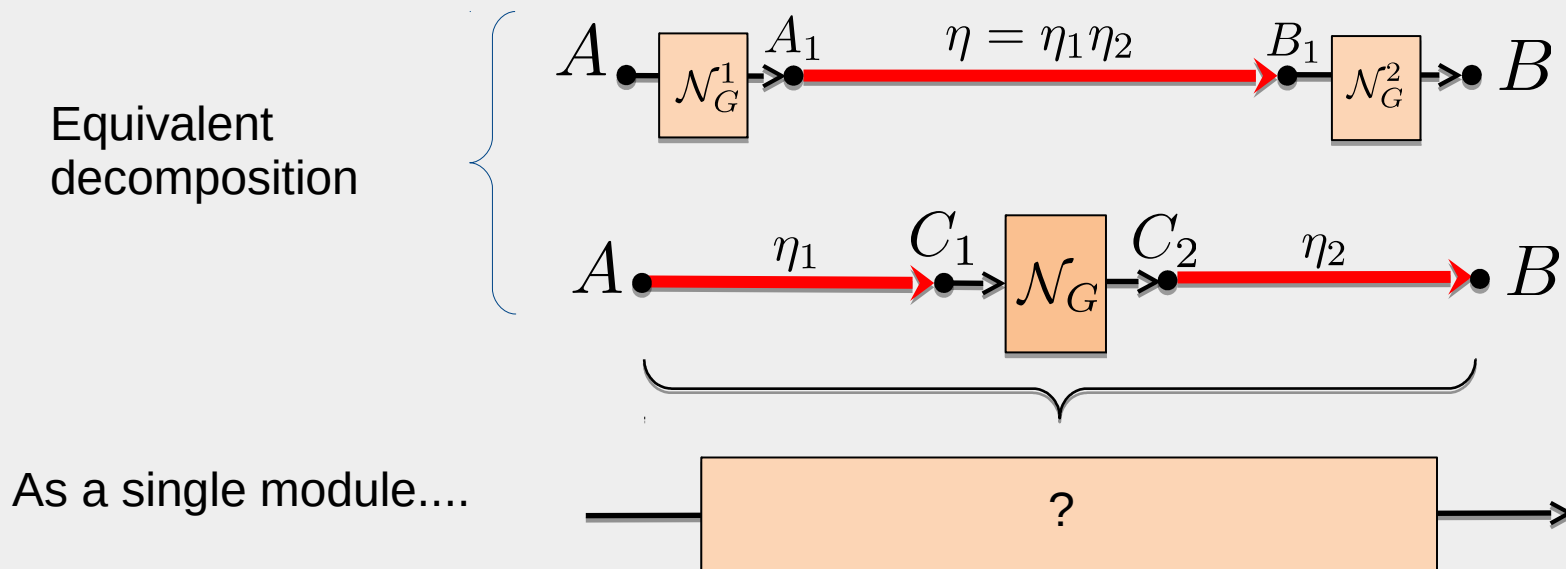
-Classical Communication

Phase insensitive amplifier (PIA) extends transmission distance



Single center station between lossy channels

- How does a center station modify the total channel?
- Single-mode Gaussian channels
- Entanglement breaking (EB) conditions



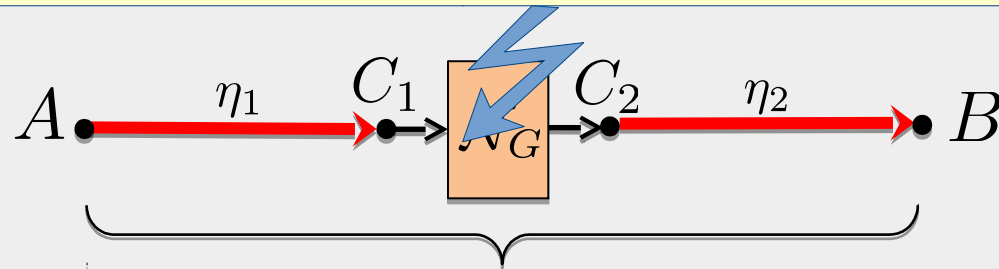
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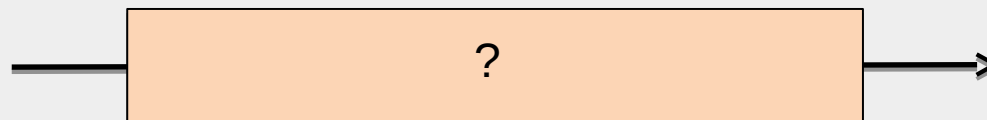
Break any quantum correlation

No entanglement / No secret key

Curty, Lewenstein & Lütkenhaus, Phys. Rev. Lett. 92, 217903 (2004).



As a single module....



Single center station between lossy channels

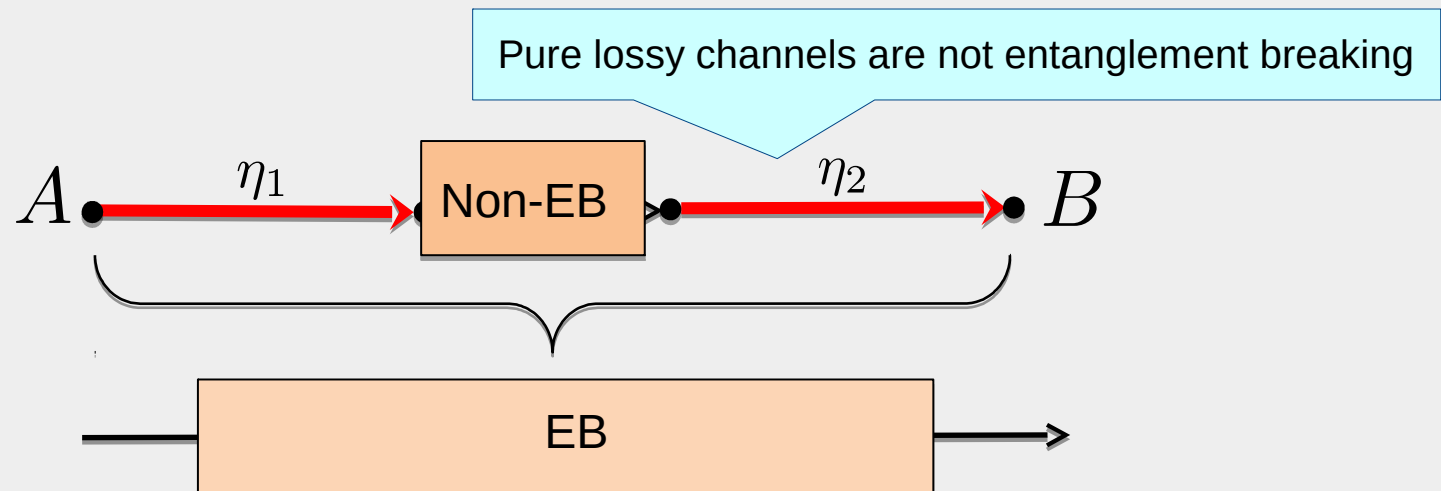
- How does a center station modify the total channel?

-Single-mode Gaussian channels

-Entanglement breaking (EB) conditions

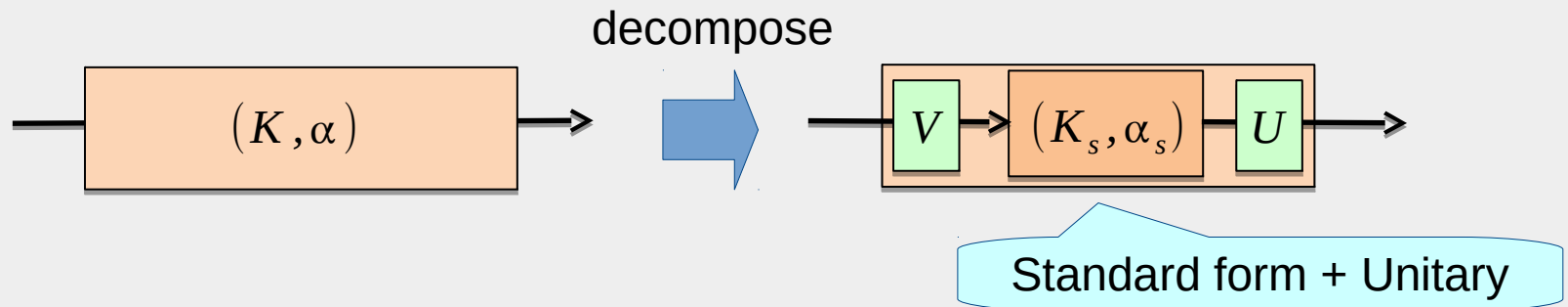
EB center station yields total EB channel.

All we have to concern about is non-EB center stations!



Single-mode Gaussian channels

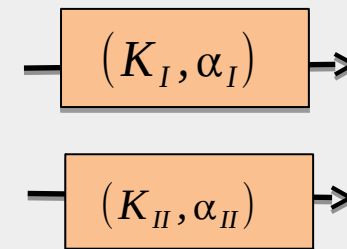
- Unitary equivalent classification



Sufficient to consider two standard forms!

(I) Phase insensitive channel (PIC)

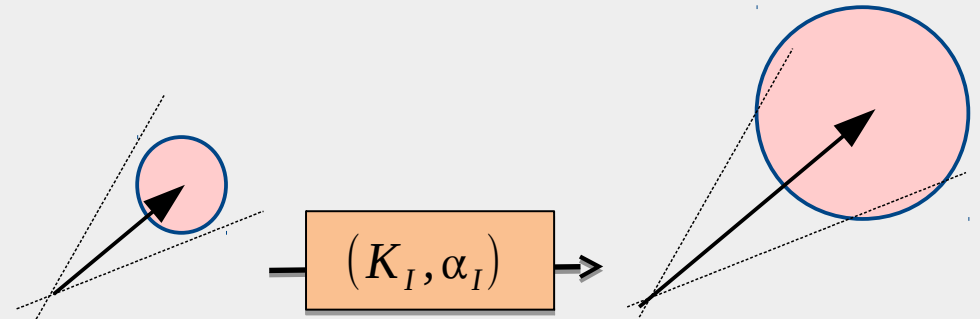
(II) Additive noise channel (ANC)



Non-entanglement breaking center stations

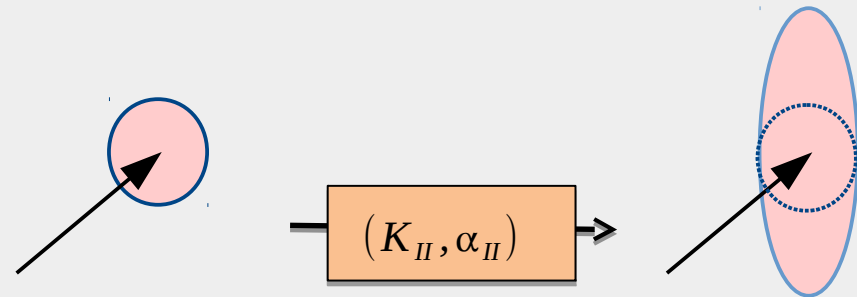
(I) Phase insensitive channel (PIC)

- Phase insensitive amplification/attenuation
- Phase insensitive noise addition



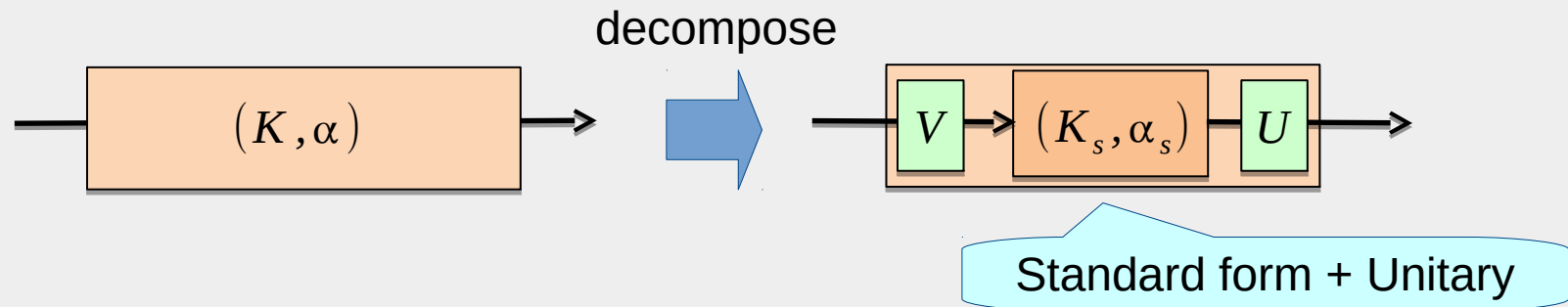
(II) Additive noise channel (ANC)

- Addition of a rank-1 noise



Single-mode Gaussian channels

- Unitary equivalent classification



Sufficient to consider two standard forms!

- Entanglement breaking (EB) conditions:

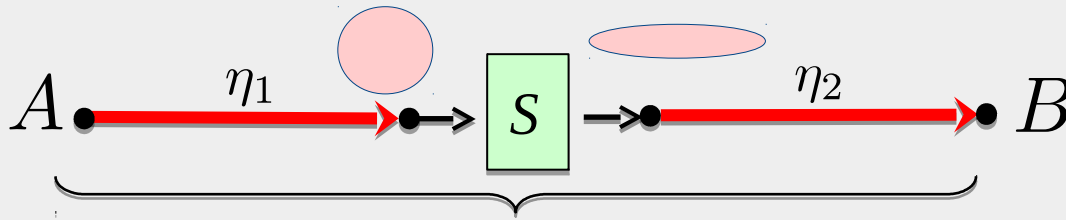
$$\sqrt{\det(\alpha_{PIC})} \geq \frac{1}{2}(1 + g \eta_1 \eta_2) \quad \text{---} (K_{PIC}, \alpha_{PIC}) \text{---} \equiv \bullet \xrightarrow{\eta_1} V \text{---} (K_I, \alpha_I) \text{---} U \xrightarrow{\eta_2} \bullet$$

$$\sqrt{\det(\alpha_{ANC})} \geq \frac{1}{2}(1 + \eta_1 \eta_2) \quad \text{---} (K_{ANC}, \alpha_{ANC}) \text{---} \equiv \bullet \xrightarrow{\eta_1} V \text{---} (K_{II}, \alpha_{II}) \text{---} U \xrightarrow{\eta_2} \bullet$$

The diagram shows two equivalent representations of entanglement breaking (EB) conditions. Each condition is shown as an inequality involving the determinant of the output covariance matrix, followed by an equivalence symbol and a block diagram. In both diagrams, an orange box represents the channel, which is equivalent to a sequence of three components: a green box V , an orange box representing the standard form (K, α) , and a green box U . Red arrows labeled η_1 and η_2 indicate the input and output states, respectively, entering and exiting the V and U boxes.

Examples: Quantum limited amplifiers

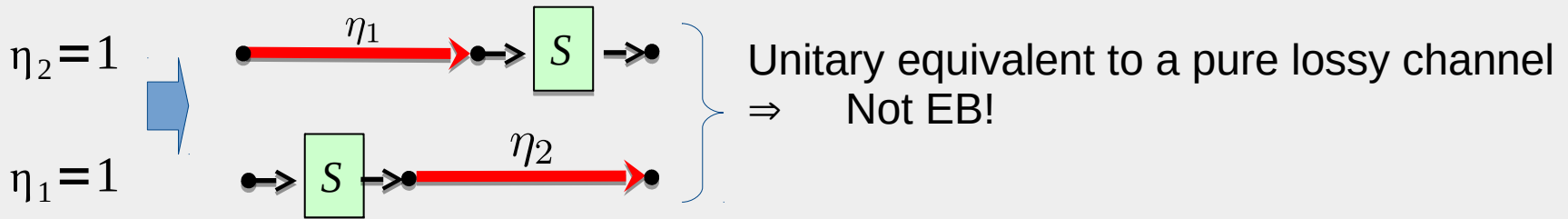
- Squeezer (Phase sensitive amplifier:PSA)



$$\frac{\Delta p^2}{\Delta x^2} = \left(\frac{\sqrt{G} - \sqrt{G-1}}{\sqrt{G} + \sqrt{G-1}} \right)^2, G \geq 1$$

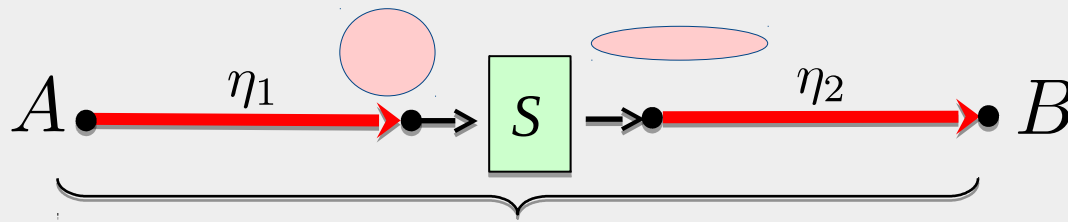
$$K = \text{Diag}[\sqrt{G} + \sqrt{G-1}, \sqrt{G} - \sqrt{G-1}]$$

$$\alpha = 0$$



Examples: Quantum limited amplifiers

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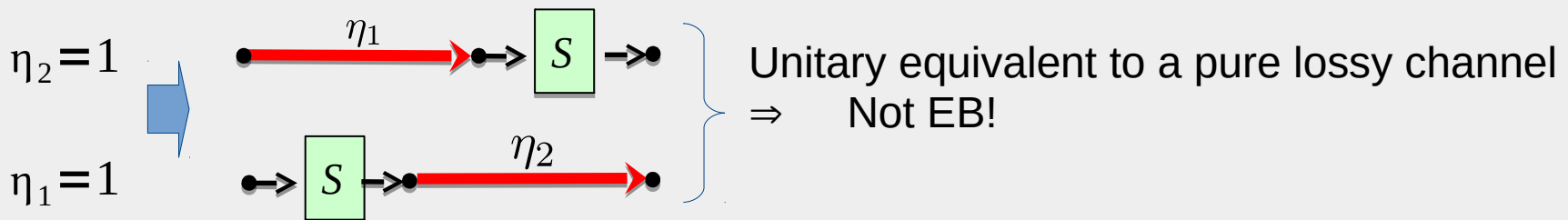
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$$\alpha = 0$$

EB if the gain fulfils

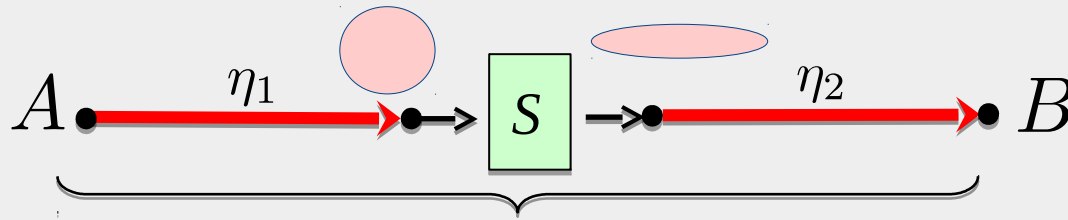
$$G_{\text{PSA}} \geq 1 + \frac{\eta_1}{(1 - \eta_1)(1 - \eta_2)}$$



A middle unitary operation renders the channel entanglement breaking!

Examples: Quantum limited amplifiers

- Squeezer (Phase sensitive amplifier:PSA)



$$\frac{\Delta p^2}{\Delta x^2} = \left(\frac{\sqrt{G} - \sqrt{G-1}}{\sqrt{G} + \sqrt{G-1}} \right)^2, G \geq 1$$

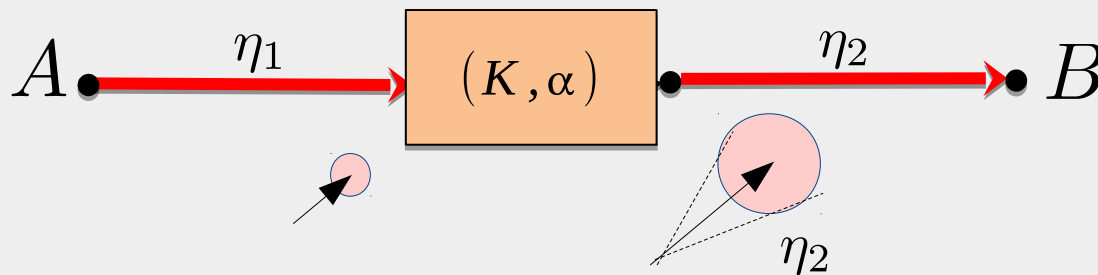
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- Quantum-limited phase insensitive amplifier (PIA)



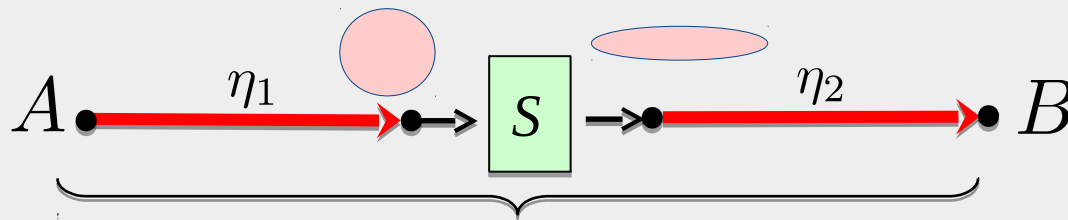
$$\frac{X_{\text{output}}}{X_{\text{input}}} = \sqrt{G}$$

$$K = \sqrt{G} I_2$$

$$\alpha = |1 - G| I_2 / 2$$

Examples: Quantum limited amplifiers

- Squeezer (Phase sensitive amplifier:PSA)



$$\frac{\Delta p^2}{\Delta x^2} = \left(\frac{\sqrt{G} - \sqrt{G-1}}{\sqrt{G} + \sqrt{G-1}} \right)^2, G \geq 1$$

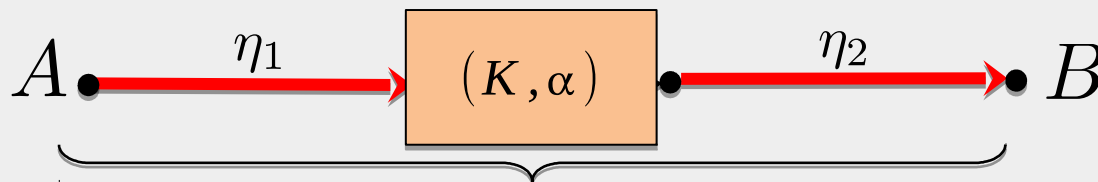
$$K = \text{Diag}[\sqrt{G} + \sqrt{G-1}, \sqrt{G} - \sqrt{G-1}]$$

$$\alpha = 0$$

EB if the gain fulfils

$$G_{\text{PSA}} \geq 1 + \frac{\eta_1}{(1 - \eta_1)(1 - \eta_2)}$$

- Quantum-limited phase insensitive amplifier (PIA)



$$\frac{X_{\text{output}}}{X_{\text{input}}} = \sqrt{G}$$

EB if the gain fulfils

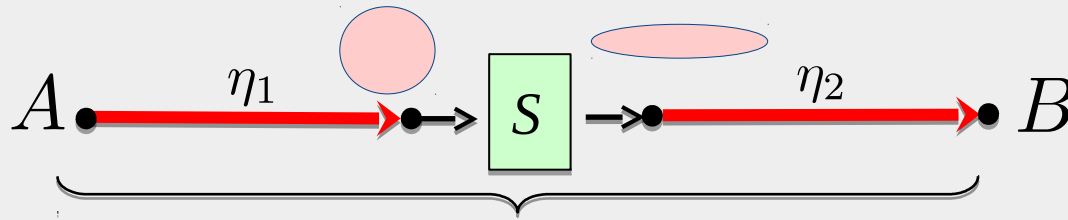
$$G_{\text{PIA}} \geq \frac{1}{1 - \eta_1}$$

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Examples: Quantum limited amplifiers

- Squeezer (Phase sensitive amplifier:PSA)



$$\frac{\Delta p^2}{\Delta x^2} = \left(\frac{\sqrt{G} - \sqrt{G-1}}{\sqrt{G} + \sqrt{G-1}} \right)^2, G \geq 1$$

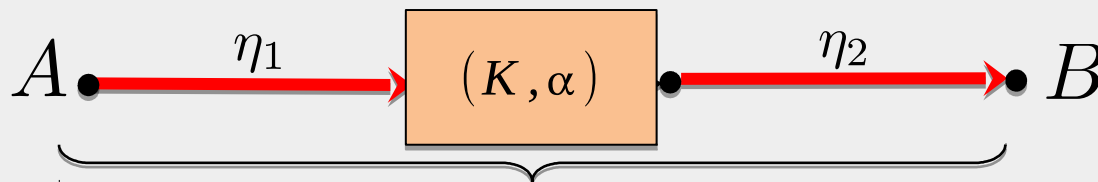
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EB if the gain fulfils

$$G_{\text{PSA}} \geq 1 + \frac{\eta_1}{(1 - \eta_1)(1 - \eta_2)}$$

- Quantum-limited phase insensitive amplifier (PIA)



$$\frac{X_{\text{output}}}{X_{\text{input}}} = \sqrt{G}$$

EB if the gain fulfils

$$G_{\text{PIA}} \geq \frac{1}{1 - \eta_1}$$

$$K = \sqrt{G} I_2$$

$$\alpha = |1 - G| I_2 / 2$$

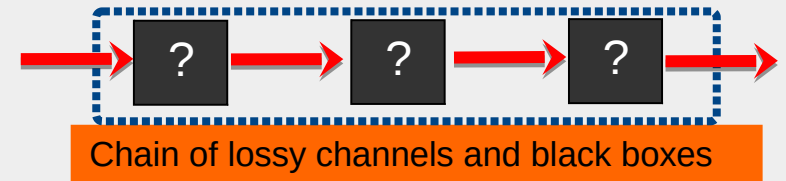
For a long distance $\eta_1, \eta_2 \ll 1 \rightarrow G \geq 1$

- a small gain amplification renders the channel entanglement breaking!

Conclusion

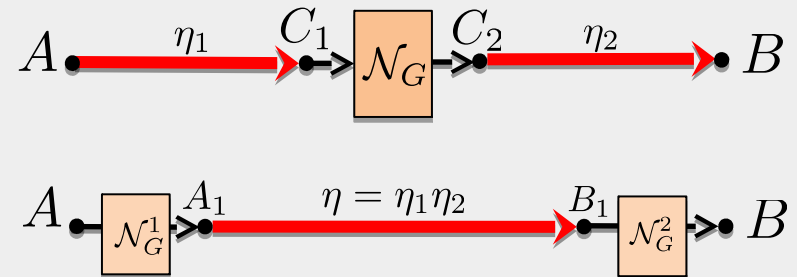
- Useful black boxes do exist!

Break the linear scaling $R \sim \eta$



- No-go result for Gaussian center stations

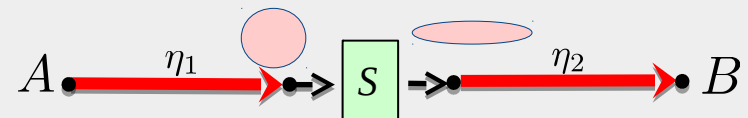
General multi-mode Gaussian channels



- Conditions that a **single-mode Gaussian station** make whole channel **entanglement breaking (EB)**

Special cases: quantum limited amplifier

- A small amplification could make the channel entanglement breaking
Better off using amplifiers as repeater stations



Construction

Sketch of Proof

- I. The total channel action
- II. Existence of a noise term and a **unitary matrix**
- III. Choose a gain term

